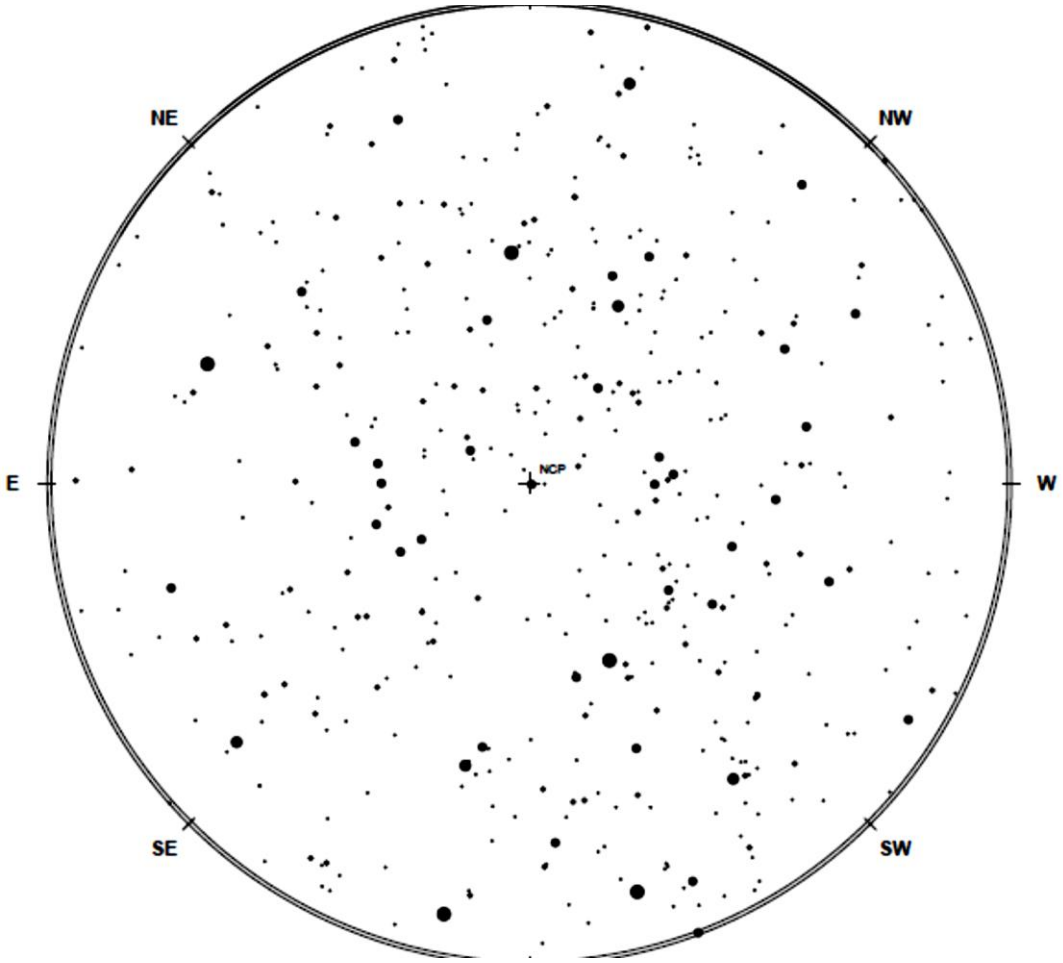
6th CAAO Problems
Senior Division

Students who are in Grade 12 or under. The winners will form the main and guest teams for the 15th IOAA, and 2 winners can be included in the main team for the 25th IAO (in β group).

1. **Coordinates** (8 points)
   a. Compute or estimate the ecliptic coordinates of α Bootes:
   (Equatorial coordinates: $\alpha = 14^h\ 16^m = 214°$ $\delta = 19°\ 11' = 19.2°$);
   b. Compute or estimate the galactic coordinates of α Lyrae:
   (Equatorial coordinates: $\alpha = 18^h\ 37^m = 280°$ $\delta = 38°\ 47' = 38.8°$).



2. **Binary Star** (10 points)

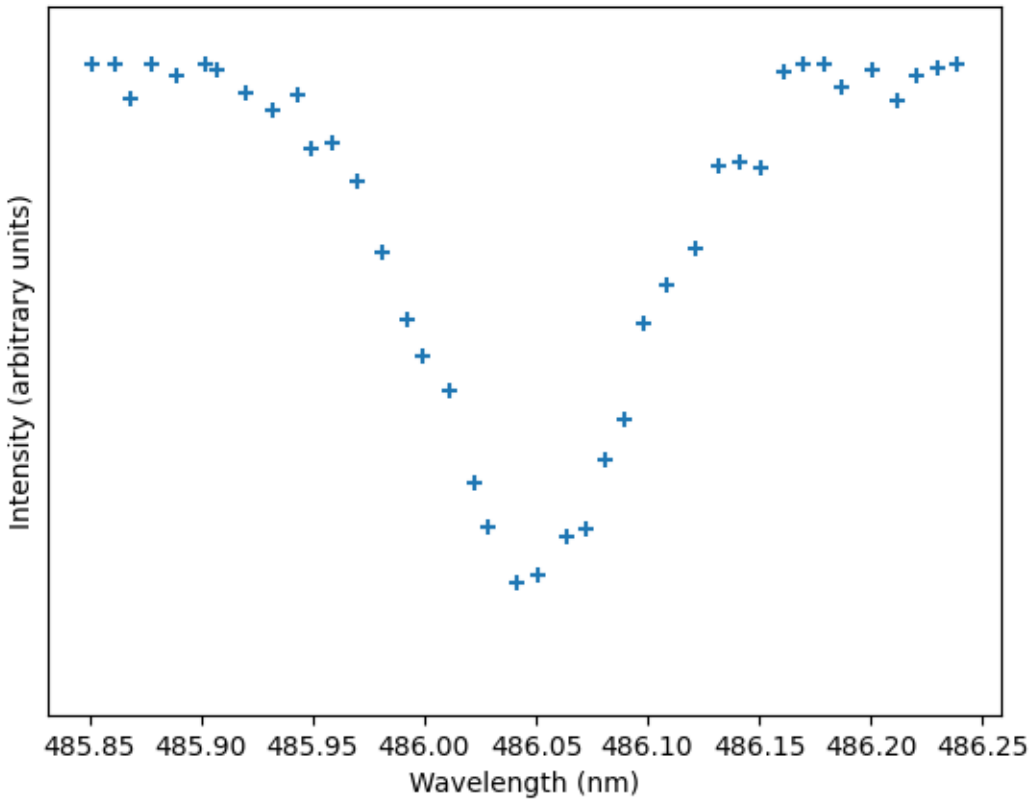
Several astronomers are studying a binary star where conservative mass transfer is occurring. Dr. Cepheid finds that mass is being transferred from a star of mass 0.86 $M_\odot$ to a star of mass 3.2 $M_\odot$. Dr. Supernova disagrees, saying her findings suggest that mass is being transferred from a star of mass 1.9 $M_\odot$ to a star of mass 1.5 $M_\odot$. Dr. Main-Sequence's findings suggest that mass is being transferred from a star of mass 2.2 $M_\odot$ to a star of the same mass. New research on the light curve of the binary shows that the orbital period of the binary star is increasing. Which of the astronomer's mass estimates could be correct? Justify your answer with appropriate physics calculations and assume orbits are circular.

3. **Galaxy** (10 points)

You just discovered a new galaxy in the Local Group! It is an edge-on spiral galaxy with apparent magnitude $m_V = 5.7$ and its disk subtends an angle $\theta = 52'$ in the sky. It has the peculiarity that it consists of identical main-sequence stars, and from analyzing the galaxy's spectrum, you notice that its peak wavelength is the same as that of the Sun. You also made some detailed measurements of the spectrum near the Hβ line, the corresponding graph is shown in the figure below. Assume the broadening of the spectral line is solely due to the variation in the velocities of the stars of the galaxy. From the galaxy's luminosity profile, you were able to determine that the number of stars per unit area of the galactic disk is approximately inversely proportional to the distance from the center of the galaxy.

A) Calculate the root-mean-square (RMS) velocity of the stars of the galaxy. Give your answer in km/s.

B) Estimate the galaxy's distance d to Earth. Give your answer in Mpc.



4. Asteroid (8 points)

An asteroid move around the Sun in an orbit with a semi-major axis equal to a = 1.078 AU and eccentricity $e = 0.826$.

Find the: average velocity, and its velocity at these three points: 1) perihelion, 2) aphelion and 3) At the point of the orbit with a true anomaly of 90◦.

Also determine the circular and escape speed at these three points of the orbit.

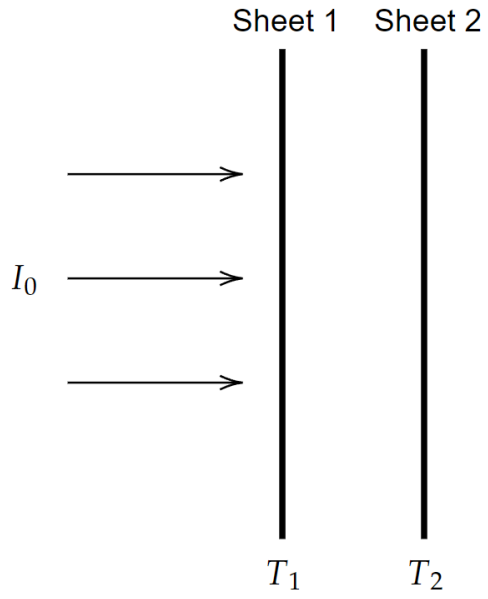
5. Redshift (8 points)

In two observed galaxies, the H-alpha spectral lines appear to be at 672.8 nm and 676.3 nm. The galaxies are 62.3° apart in the sky. Find the redshift of one galaxy from the perspective of the other.

The rest wavelength of H-alpha is 656.5 nm and the Hubble's constant is 72 km/s/Mpc. Use non-relativistic formulas.

6. JWST (8 points)

An integral component of the James Webb Space Telescope is its sunshield, which protects its infrared telescope from the heat and radiation of the Sun. In this problem, we'll study a simplified model of a sunshield with two sheets of emissivity ε. Assume that radiation is the main means of heat transfer between the sheets and neglect radiation leaking out the sides. The radiation from the Sun is perpendicular to the sunshield with intensity I_0, and you may neglect other sources of radiation such as the CMB.



(a) Imagine a light source placed in between the two sheets. The source emits a ray of light towards sheet 2, perpendicular to the sheet. What fraction of the light gets absorbed by sheet 2?

(b) Using the result from part (a), compute the temperatures T_1 and T_2 of the sheets. Find the minimum possible value of T_2 and compute its numerical value using a distance of 1.010 AU from the Sun. Is it low enough for the infrared telescope to function? (The infrared telescope can only function below 40 K.)

Junior Division

Students who are 15 (born after January 1, 2007) or younger. The winners can be included in the main team at the 25th IAO (in α group) and the guest team at the 15th IOAA.

1. **Constellation** (8 points)

   A failure in a spacecraft's control system changed the thrust directions of its three engines. The thrust of the first engine is now directed towards Cygnus and the thrust of the second towards Cancer. To which constellation is the thrust of the third engine directed if the total thrust is zero? The power of all three engines is equal.

2. **Orbit Transfers** (8 points)

   Describe how can the speed of an artificial satellite be changed in order to transfer it from a circular orbit to an orbit with an eccentricity of 0.4? Consider all possible options.

3. **Moon** (8 points)

   At what geographical latitudes can the Moon be observed at its zenith?

4. **Planet** (8 points)

   A planet takes ΔT=390 days to go from conjunction to opposition. Find the change in magnitude Δm between the two events. Assume the orbits of the planet and the Earth are circular, lying in the same plane. Name the planet.

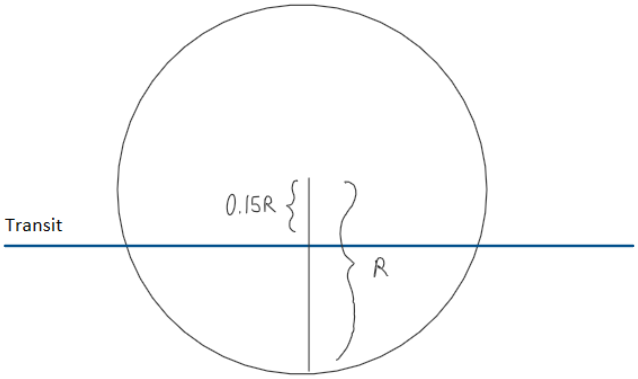
5. **Redshift** (8 points)

   In two observed galaxies, the H-alpha spectral lines appear to be at $672.8\ nm$ and $676.3\ nm$. The galaxies are 62.3° apart in the sky. Find the redshift of one galaxy from the perspective of the other.

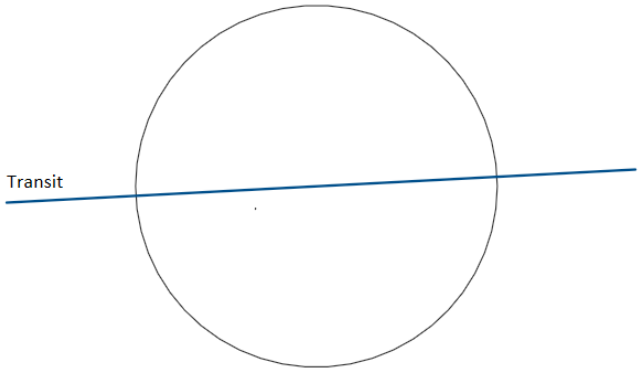
   The rest wavelength of H-alpha is $656.5\ nm$ and the Hubble's constant is $72\ km/s/Mpc$. Use non-relativistic formulas.

6. **Transits** (10 points)

Two observers on different planets see the same object transiting a nearby star with transit paths shown below. The first is a horizontal line, and the second passes through the center but is inclined 0.24°. The angle formed by the first observer, the star, and the second observer is 90°, and the observers are inclined the same way (they stand on parallel surfaces). If the star's mass is equal to the sun's mass, and the transit time from the second observer's perspective was 4 hours 32 minutes, find the radius of the star.



Transit from First Observer's Perspective



Transit from Second Observer's Perspective

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The following sources were used in preparations of problems:
1. Tasks of the Moscow Astronomical Olympiad 1997-2002
2. A.P. Klishenko, V.I. Shuplyak. Astronomy. Tutorial. 2004
3. V.Ignatiev Problems in astronomy and astrophysics. 2021