Canadian Astronomy Olympiad Preparation

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Basic Concepts

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Preface

This tutorial is written for the students who want to participate in the Canadian Astronomy Olympiad (CAO). This document is only an introductory level guide for the interested students, and its purpose is to introduce the Astronomy Olympiad to every student in Canada who wants to participate in it. Every year, the top CAO students are selected to represent team Canada in the International Astronomy and Astrophysics Olympiad (IOAA). The selected team will go through a training program to be ready for the international competition.

While this tutorial has a lot of information in it, the students are encouraged to supplement it with other textbooks that are named in the resources section. There are numerous “Practice Problems” within this document that are designed for the students to be a complement on their learning path. In addition to that, it is highly recommended to solve the past CAO problems that are provided in a separate PDF file.

As a prerequisite, the Canadian astronomy olympiad would require high school level physics and mathematics; however, for the international Olympiads, students would need to learn beyond high school level physics and mathematics.

The International Astronomy and Astrophysics Olympiad (IOAA) is a highly reputable international competition at the high school level; every year the elite students of each country participate in this Olympiad; Canada has been participating in IOAA since 2013.

Resources

There are a few textbooks that can be helpful in preparation for the CAO. Some suggested resources are listed below:

1) *Foundations of Astrophysics* by Barbara Ryden
2) *Fundamental Astronomy* by Karttunen et. al.
3) *An Introduction to Modern Astrophysics* by Bradley Carroll and Dale Ostlie
4) *Astronomy principles and practice* by Archie E. Roy and David Clarke
5) *Introduction to Cosmology* by Barbara Ryden

The first two textbooks are introductory level, if students have a solid physics and calculus background, they can start by reading *An Introduction to Modern Astrophysics*. The topic of spherical astronomy is best discussed in the *Astronomy principles and practice*, so this book is necessary to learn spherical astronomy.

In addition to these books, students should be confident in high school physics and mathematics. For the senior level students, it is highly recommended to learn calculus as well.

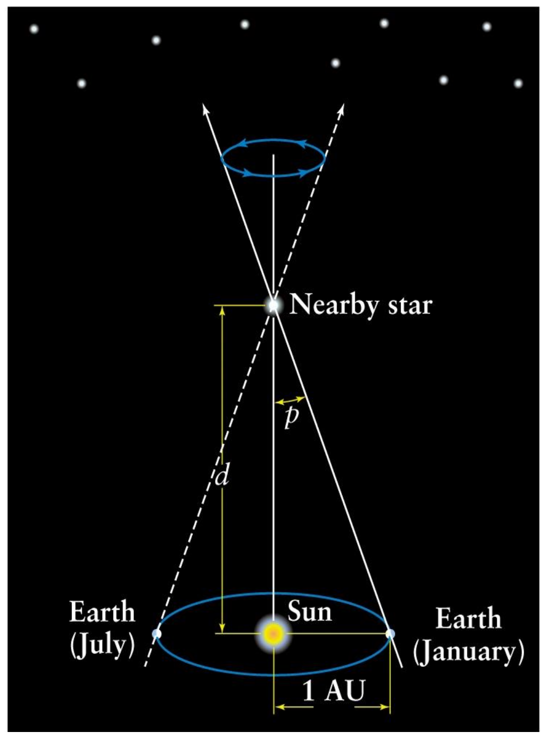
For more advanced IOAA level resources you can checkout the references that was used to write this document. They are all written at the reference section so that the enthusiastic students might search for.

Parallax

Measuring the intrinsic brightness of stars is linked with determining their distances. On Earth, the distance to the peak of a remote mountain can be determined by measuring that peak's angular position from two observation points separated by a known baseline distance. Simple trigonometry then supplies the distance to the peak. Finding the distance even to the nearest stars requires a longer baseline. As Earth orbits the Sun, two observations of the same star made 6 months apart employ a baseline equal to the diameter of Earth's orbit. These measurements reveal that a nearby star exhibits an annual back-and-forth change in its position against the stationary background of much more distant stars. a measurement of the parallax angle p (one-half of the maximum change in angular position) allows the calculation of the distance d to the star.

We can write the equation as:

$$d = \frac{1\,AU}{\tan p} \approx \frac{1}{p} AU \qquad \text{Equation 1}$$



The angle p is smaller as distance becomes larger. Using parallax, we are going to introduce a new distance measure, *parsec*.

A parsec is the distance at which 1 Astronomical Unit subtends an angle of 1 second of arc (arcsecond).

$$1\,parsec \;=\; 3.26\,light\ years$$

$$1pc = \frac{1\,AU}{\frac{1}{3600} \times \frac{\pi}{180}} = 206265\,AU$$

1 arc second is $1/60$ of an arc minute, and an arc minute is $1/60$ of one degree. Therefore, an arc second is $1/3600^{\text{th}}$ of one degree.

Stellar Luminosity

Stars are considered as a spherical source radiating energy due to its temperature; its total energy output can be determined by equation below, according to its surface temperature and its surface area. This total output is referred to as the **stellar luminosity**, L, and may be expressed as

$$L = 4\pi R^2 \sigma T^4 \qquad \text{Equation 2}$$

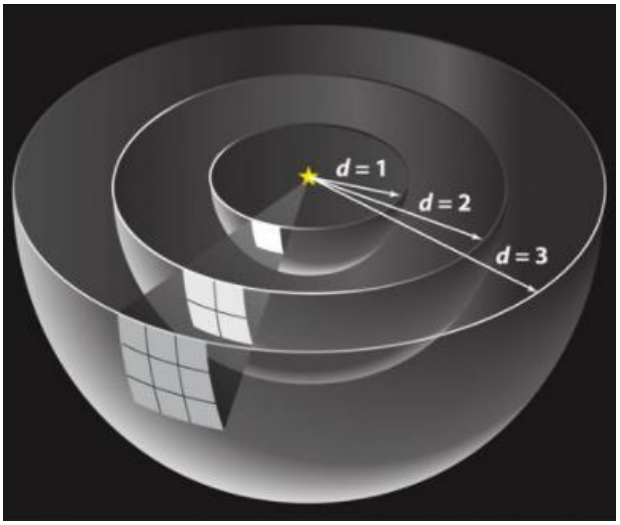
where R is the radius of the star, σ is known as **Stefan's constant** and T is the star's surface temperature. The unit of luminosity is Watts (joules per second). For instance, the luminosity of our sun is $3.85 \times 10^{26}\ W$.

Brightness (radiant flux)

The brightness of a star is measured in terms of the flux received from the star. The power received per unit area at the Earth depends on the stellar luminosity and on the inverse square of the stellar distance. If the latter is known, the flux provided by the source may be readily calculated and expressed in terms of watts per square metre ($^{W}/_{m^2}$).

Imagine a star of luminosity L surrounded by a huge spherical shell of radius d. Then, assuming that no light is absorbed during its journey out to the shell, the radiant flux, b, measured at distance d is related to the star's luminosity by

$$b = \frac{L}{4\pi d^2} \qquad \text{Equation 3}$$



the denominator being simply the area of the sphere. Since L does not depend on d, the radiant flux is inversely proportional to the square of the distance from the star. This is the well-known inverse square law for light.

Practice Problem 1

The luminosity of the Sun is $L = 3.85 \times 10^{26}\ W$. If the distance of the Earth from Sun is equal to $1\ AU$. Determine what is the flux that Earth receives from Sun.($1\ AU = 1.496 \times 10^{11}\ m$)

Magnitude system

Invented by the astronomer Hipparchus 2200 years ago, it was simply a way to "rank" the stars visible at night. The brightest were ranked as 1st magnitude, the faintest visible were ranked as 6th magnitude. In other words, the brightest stars were assigned the smallest number, the faintest the largest number. And 6 divisions were used because of the mysticism about 6, which is the first "perfect number."

Brightness ratio of rank first and sixth is 100:

$$K^5 = 100 \rightarrow K = \sqrt[5]{100} = 2.5118 \rightarrow \frac{b_2}{b_1} = 2.5118^{m_1 - m_2}$$

$$m_1 - m_2 = -2.5 \, log \frac{b_1}{b_2} \qquad \text{Equation 4}$$

Where, m is the magnitude of the stars. The magnitude system is based on comparison; This means that you need to know the magnitude of a certain star and comparing its brightness with other stars you can determine the magnitude of the star.

Practice Problem 2

> Determine the brightness of a star with the magnitude of 2. You can use Sun as the known star to compare. (The apparent magnitude of our Sun is -26.8)

The absolute magnitude, M, is defined to be the apparent magnitude a star would have if it were located at 10 pc. Recall that a difference of 5 magnitudes between the apparent magnitudes of two stars corresponds to the smaller magnitude star being 100 times brighter than the larger-magnitude star. This allows us to find an equation for the absolute magnitude just like the apparent magnitude:

$$M_1 - M_2 = -2.5 \, log \frac{L_1}{L_2} \qquad \text{Equation 5}$$

Where M is the absolute magnitude and L is the luminosity of the star. To use this equation, we need to know a specific star's absolute magnitude and luminosity to be able to compare it with other stars. We have defined two different magnitudes, absolute and apparent. We defined absolute magnitude as the apparent magnitude at a certain distance ($10 parsecs$), therefore, there should be a connection between the star distances and their magnitudes. This relation is called *Distance Modulus:*

$$m - M = 5 \, log \, d - 5 \qquad \text{Equation 6}$$

Where d is in parsecs, m and M are apparent and absolute magnitudes respectively. Unlike the previous magnitude relations (*Equations 4-5*), distance modulus is written for a single star. It is the relationship between the absolute and apparent magnitude of any star in the distance d. Using this relation, it is clear that if you have any star in the distance of 10 parsecs, then the two magnitudes should be equal to each other.

Limiting magnitude

Looking up at the night sky, we are not able to see all the stars; our eyes have limited light gathering aperture (around 6mm in a dark night!). The faintest stars that your naked eye can see in the night is about $6 - 6.5$ magnitudes. However, this limiting magnitude might also change due to light pollution or atmospheric effects. For instance, in a metropolitan area your limiting magnitude might go up to 2-3 magnitudes. This would seriously limit your ability to see constellations.

We know that using optical devices would enable us to see fainter objects in the sky. For instance, if you are using a telescope since it has a bigger diameter, it is able to gather more light than your eyes can. Therefore, we have the equation below to determine the limiting magnitude of a telescope:

$$m_e - m_t = -5 \, log \frac{D_t}{D_e} \qquad \text{Equation 7}$$

Where m_e is the limiting magnitude of naked eye ($m_e \approx 6.5$), m_t is the telescopes limiting magnitude, D_e is the eye's diameter ($D_e \approx 6mm$) and D_t is the telescopes diameter.

Practice Problem 3

Determine the limiting magnitude of an 8-inch telescope.

Optical Telescopes

An optical telescope forms images of faint and distant stars. It can collect much more light from space than the human eye can. Optical telescopes are built in two basic designs—**refractors** and **reflectors**. The heart of a telescope is its objective, a main lens (in refractors) or a mirror (in reflectors). Its function is to gather light from a sky object and focus this light to form an image. The ability of a telescope to collect light is called its **light-gathering power**.

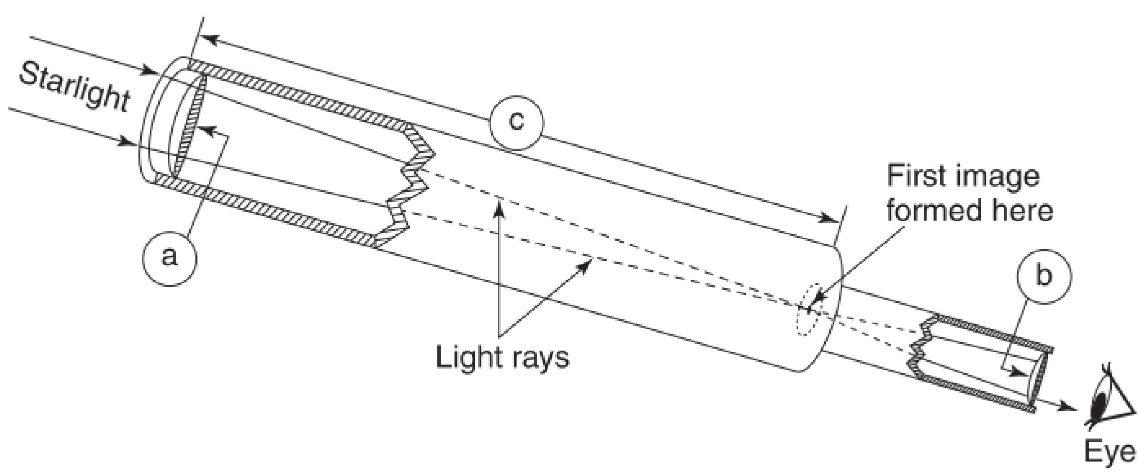
Light-gathering power is proportional to the area of the collecting surface, or to the square of the **aperture** (clear diameter of the main lens or mirror). The **size** of a telescope, such as 150-mm or 8-m (6 inch or 26-foot), refers to the size of its aperture. You can look at the image directly through an eyepiece, which is essentially a magnifying glass. Or you can photograph the image or record and process it electronically. Your eye lens size is about 6 mm. A 150- mm (6-inch) telescope has an aperture over 30 times bigger than your eye lens. Its light-gathering power is 900 times greater than that of your eye. So, a star appears over 900 times brighter with a 150-mm (6-inch) telescope than it does to your unaided eye.

Astronomers build giant telescopes to detect fainter and more distant objects. All stars appear brighter with telescopes than they do to the eye alone. The extra starlight gathered by the telescope is concentrated into a single point. Using time exposure, a giant 10-m (400-inch) telescope can image very faint stars down to about magnitude 28, which is the same apparent brightness as a candle viewed from the Moon!

Refracting Telescopes

A refracting telescope has a main, objective lens permanently mounted at the front end of a tube. Starlight enters this lens and is refracted, or bent, so that it forms an image near the back of the tube. The distance from this lens to the image is its focal length. You may look at the image through a removable magnifying lens called the eyepiece. The tube keeps out scattered light, dust, and moisture. Italian astronomer Galileo Galilei (1564–1642) first pointed a refracting telescope skyward in 1609. The largest instrument he made was smaller than 50 mm (2 inches).

Today refracting telescopes range in size from a beginner's 60-mm (2.4-inch) to the largest ever built, the 1-m (40-inch) telescope at the Yerkes Observatory in Williams Bay, Wisconsin, U.S., which was completed in 1897.



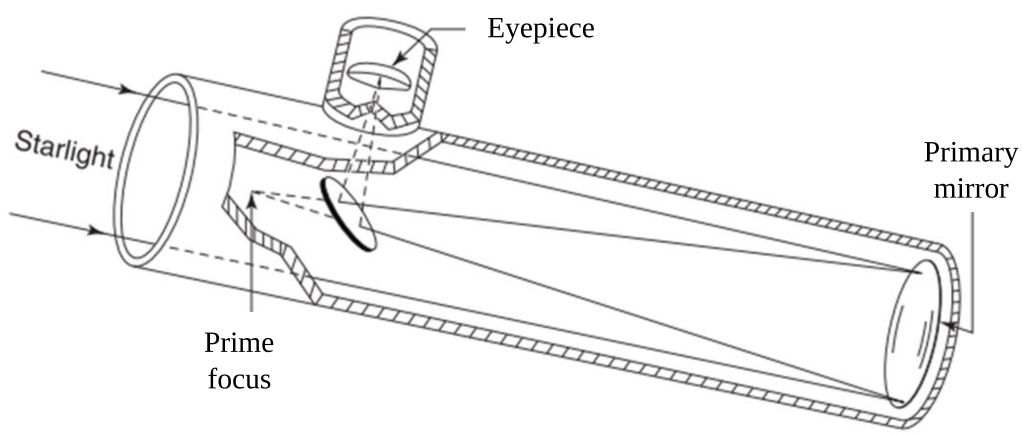
In the figure above: (a) Objective lens gathers the light and form an image. (b) Eyepiece magnifies the image formed by the objective. (c) Focal length of objective lens.

Reflecting Telescopes

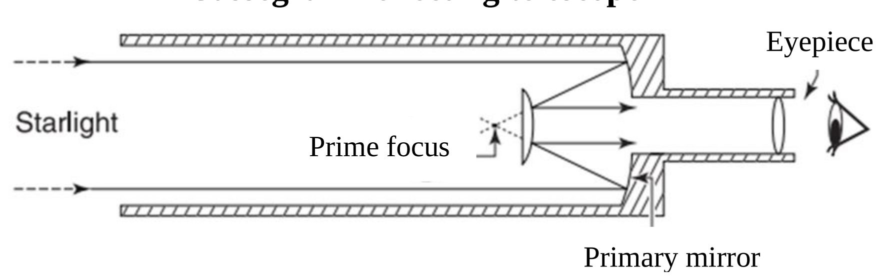
A **reflecting telescope** has a highly polished curved-glass mirror, the **primary mirror**, mounted at the bottom of an open tube. When starlight shines on this mirror, it is reflected back up the tube to form an image at the **prime focus**.

You can record the image at the prime focus, or you can use additional mirrors to reflect the light to another spot. The **Newtonian telescope**, originated by British scientist Sir Isaac Newton in 1668, uses a small, flat mirror to reflect the light through the side of the tube to an eyepiece (Figure below). The **Cassegrain telescope** uses a small convex mirror, a **secondary mirror**, to reflect the light back through a hole cut in the primary mirror at the bottom end of the tube. It is more compact than a refractor or Newtonian reflector of the same aperture. The **Schmidt-Cassegrain** telescope combines an extremely short-focus spherical primary mirror at the back end of a sealed tube with a thin lens at the front.

Newtonian reflecting telescope



Cassegrain reflecting telescope



F number

Telescopes are often described by both their aperture size and f number. The **f-number** is the ratio of the focal length of the main lens or mirror to the aperture. These specifications are important because the brightness, size, and clarity of the image produced by a telescope depend on the aperture and focal length of its main lens or mirror. For example, a “150-mm (6-inch), f/8 reflector” means the primary mirror is 150 mm (6 inches) in diameter and has a focal length of 1200 mm (8 × 150), or 48 inches (8 × 6).

Images

All stars except our Sun are so far away that they appear as dots of light in a telescope. The Moon and planets appear as small disks. **Image size** is proportional to the focal length of the telescope’s main lens or mirror.

For example, a mirror with a focal length of 2.5 m (100 inches) produces an image of the Moon that measures about 2.5 cm (1 inch) across. You know that the 5-m (200-inch), f/3.3 mirror has a focal length of 16.5 m (660 inches), which is over six times as long. Hence, it produces an image of the Moon that is about six times as big, or 15 cm (6 inches) across. Lenses and mirrors form real images that are upside down. (A real image is formed by the actual convergence of light rays.) Since inverted images do not matter in astronomical work and righting them would require additional light-absorbing optics, nothing is done to turn images upright in telescopes.

Telescope resolving power

The **resolving power** of a telescope is defined as the ability of a telescope to distinguish objects with a small angle between them. This is known as the **theoretical resolving power** of the instrument. If the telescope is of good design and in correct adjustment, it should be possible to achieve this theoretical value.

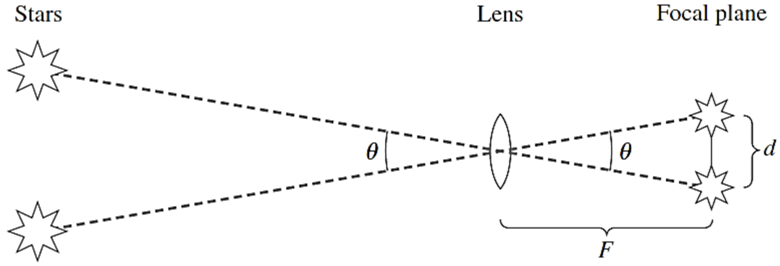
It should be possible to resolve two stars if they are separated by an angle (in radians) greater than

$$\alpha = 1.22\frac{\lambda}{D} \qquad \text{Equation 8}$$

This value is known as the **theoretical angular resolving power** of the telescope. It can be seen that the resolving power is inversely proportional to the diameter of the objective. We take a value of $5500\ Angstroms$ for λ being the effective wavelength or visual observations.

Lens and the focal length

In a camera with a lens, the image will be in focus only at a fixed distance F from the lens. The distance F to the **focal plane** depends on the shape of the lens, as well as on its refractive index. For lenses, a useful parameter is the focal ratio $f = F/D$, where D is the diameter of the lens. The size of the image produced is not affected by the diameter D of the lens but only by the focal length F.



In the figure above, two stars separated by a small angle θ on the sky have images that are separated by a physical distance d on the focal plane. Another useful parameter, in addition to the focal length, is the scale of the image on the focal plane, known for historical reasons as the **plate scale**. Specifically, an angular distance θ on the celestial sphere is related to a physical distance d on the image plane by the plate scale s:

$$\theta[arcsec] = s[arcsec/mm]\,.\,d[mm] \qquad \text{Equation 9}$$

We can also write:

$$\theta[radians] = \frac{d}{F} \qquad \text{Equation 10}$$

And therefore:

$$\theta[arcsec] = \theta[radians].\frac{180°}{\pi\, radians}.\frac{3600\, arcsec}{1°} = 206{,}265\ (\frac{d}{F}) \qquad \text{Equation 11}$$

we have a relationship between the plate scale s and the focal length F:

$$s[arcsec/mm] = \frac{206{,}265}{F[mm]} \qquad \text{Equation 12}$$

The human eye, for instance, has a focal length $F \approx 17\ mm$, and hence a “plate scale” $s \approx 12{,}100\ arcsec/mm$, or $s \approx 3.4°/mm$; when you look at the full Moon, its image covers an area of your retina less than $0.15mm$ across.

Large astronomical telescopes have focal lengths that are more conveniently expressed in meters than in millimeters. For these big telescopes, we may write

$$s[arcsec/mm] = \frac{206.265}{F[m]} = \frac{206.265}{fD[m]} \qquad \text{Equation 13}$$

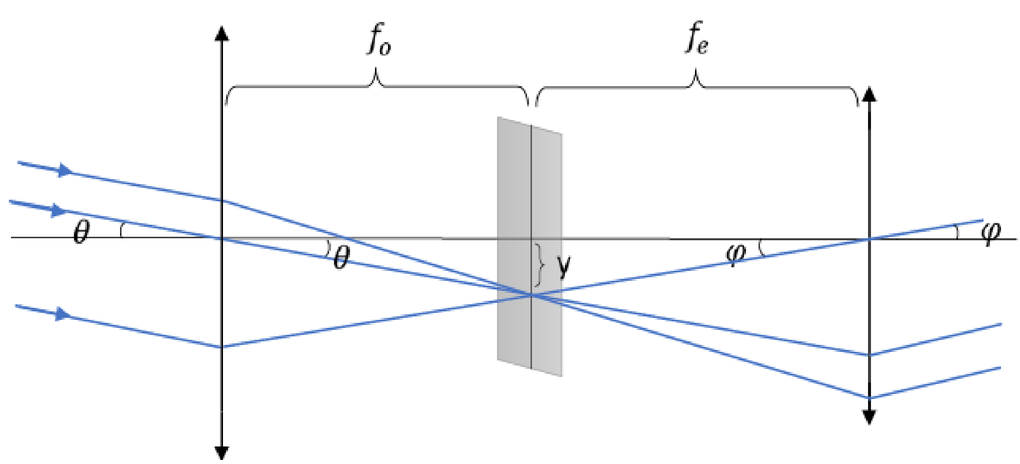
where f is the focal ratio, and D is the diameter of the telescope's aperture. As an example, the famous "forty-inch" Yerkes Telescope (at Williams Bay, Wisconsin) has an aperture $D = 1.02\ m$ and a focal ratio $f = 19$. The plate scale of the Yerkes Telescope is thus:

$$s = \frac{206.265}{19(1.02)}\ arcsec/\ mm = 10.6\ arcsec/\ mm$$

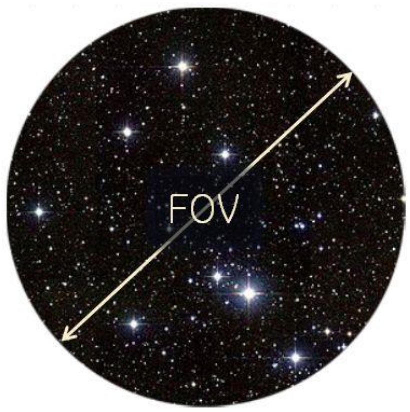
An image of the full Moon produced by the Yerkes Telescope is therefore 170 mm across, about the size of a salad plate.

The major optical component of a refracting telescope is the primary or objective lens of focal length f_{obj}. The purpose of the objective lens is to collect as much light as possible and with the greatest possible resolution, bringing the light to a focus at the focal plane. A photographic plate or other detector may be placed at the focal plane to record the image, or the image may be viewed with an eyepiece, which serves as a magnifying glass. The eyepiece would be placed at a distance from the focal plane equal to its focal length, f_{eye}, causing the light rays to be refocused at infinity. Figure below shows the path of rays coming from a point source lying off the optical axis at an angle θ. The rays ultimately emerge from the eyepiece at an angle φ from the optical axis. The angular magnification produced by this arrangement of lenses can be shown to be:

$$m = \frac{f_{obj}}{f_{eye}} \qquad \text{Equation 14}$$



In astronomy, the field of view is the amount of sky you can see, whether with your unaided vision, binoculars, or a telescope. If you had eyes on all sides of your head, you would have a 360° field of view. (Some insects actually do!) If you include peripheral vision, your naked eye field of view is nearly 180°, but with varying quality across this field. A telescope will have a much smaller field of view, but it has significant advantages, such as greater magnification and light-gathering power.



Field of view (FOV) is the diameter of a region of the sky that you can see using a specific instrument. The FOV would change with the magnification of the telescope you are using.

Orbital Mechanics

Isaac Newton (1642/3–1727) was born in rural England. When young Newton proved to be incompetent at managing his family's farm, he was sent to Cambridge University and started to thrive as a scholar. In 1665, the year in which Newton earned his bachelor's degree, an outbreak of the plague closed down the university, and Newton retreated to his family's farm and began to think—very hard. The period when the university was closed was Newton's *annus mirabilis*, during which he discovered calculus, formulated his three **laws of motion** and his **law of universal gravitation**, and performed ground-breaking experiments in optics. Newton's law of universal gravitation can be concisely expressed in mathematical form. Suppose that two spherical objects of mass M and m, are separated by a distance r. Newton's law tells us that the gravitational attraction between the two objects is:

$$F = -\frac{GMm}{r^2} \quad \text{Equation 15}$$

where G is the **gravitational constant**, is a universal constant whose value is $G = 6.67 \times 10^{-11}\, Nm^2kg^{-2}$ (where N stands for newton). The negative sign in equation above tells us that gravity is always an attractive force. Johannes Kepler, working with data painstakingly collected by Tycho Brahe without the aid of a telescope, developed three laws which described the motion of the planets across the sky.

1. The Law of Orbits: All planets move in elliptical orbits, with the sun at one focus.

2. The Law of Areas: A line that connects a planet to the sun sweeps out equal areas in equal times.

3. The Law of Periods: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

Within the solar system, if the size of the orbit (a) is expressed in astronomical units (1 AU equals the average distance between the Earth and Sun) and the period (P) is measured in years, then Kepler's Third Law says:

$$P^2 = a^3 \qquad \text{Equation 16}$$

After applying Newton's Laws of Motion and Newton's Law of Gravity we find that Kepler's Third Law takes a more general form:

$$P^2 = \left(\frac{4\pi^2}{G(m_1+m_2)}\right) a^3 \qquad \text{Equation 17}$$

Where m_1 and m_2 are the masses of the two bodies. Let's assume that one body, m_1 say, is always much larger than the other one. Then $m_1 + m_2$ is nearly equal to m_1.

Doppler effect and Red Shift

In 1842 the Austrian physicist Christian Doppler showed that as a source of sound moves through a medium (such as air), the wavelength is compressed in the forward direction and expanded in the backward direction. This change in wavelength of any type of wave caused by the motion of the source or the observer is called a Doppler shift. Doppler deduced that the difference between the wavelength λ_{obs} observed for a moving source of sound and the wavelength λ_{rest} measured in the laboratory for a reference source at rest is related to the radial velocity v_r (the component of the velocity directly toward or away from the observer, of the source through the medium by:

$$\frac{\lambda_{obs}-\lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v_r}{v_s} \quad \text{Equation 18}$$

where v_s is the speed of sound in the medium. When astronomers observe a star or galaxy moving away from or toward Earth, the wavelength of the light they receive is shifted toward longer or shorter wavelengths, respectively. If the source of light is moving away from the observer ($v_r > 0$), then $\lambda_{obs} > \lambda_{rest}$. This shift to a longer wavelength is called a **redshift**. Similarly, if the source is moving toward the observer ($v_r < 0$), then there is a shift to a shorter wavelength, a **blueshift**. most of the objects in the universe outside of our own Milky Way Galaxy are moving away from us, redshifts are commonly measured by astronomers. A redshift parameter z is used to describe the change in wavelength; it is defined as

$$z = \frac{\lambda_{obs}-\lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} \quad \text{Equation 19}$$

There is a relation between the velocity of the object moving away or toward us and the redshift z. If the velocity of the object is comparable to the speed of light, we need to use the relativistic formula:

$$z = \sqrt{\frac{1+\frac{v_r}{c}}{1-\frac{v_r}{c}}} - 1 \quad \text{Equation 20}$$

Where c is the speed of light. Its non-relativistic formula can be written as:

$$z = \frac{v_r}{c} \quad \text{Equation 21}$$

Hubble's Law

In the late 1920's, Hubble discovered that the spectral lines of galaxies were shifted towards the red by an amount proportional to their distances. If the redshift is due to the Doppler effect, this means that the galaxies move away from each other with velocities proportional to their separations (this means that the Universe is expanding as a whole). Hubble's law states that the redshift in light coming from distant galaxies is proportional to their **distance**. The discovery of the linear relationship between recessional velocity and distance yields a straightforward mathematical expression for Hubble's Law as follows:

$$v = H_0 D \qquad \text{Equation 22}$$

where v is the recessional velocity due to redshift, typically expressed in km/s, D is the distance of the object in Mpc, and H_0 is Hubble's constant. Usually, the Hubble's constant is around $65 - 75\ km/s.Mpc$ in the problems.

Practice Problem 4

The **Ca II, H** and **K** lines have rest wavelengths of $\boldsymbol{\lambda_{rest}} = 3968.5\,A$ and $3933.6\,A$ respectively. In the spectrum of a galaxy in the cluster Abell 2065 (a.k.a. the Corona Borealis Cluster), the observed wavelengths of the two lines are $\boldsymbol{\lambda_{obs}} = 4255.0\,A$ and $4217.6\,A$ respectively.

1. What is the redshift z of the galaxy?
2. What is the distance to the galaxy?

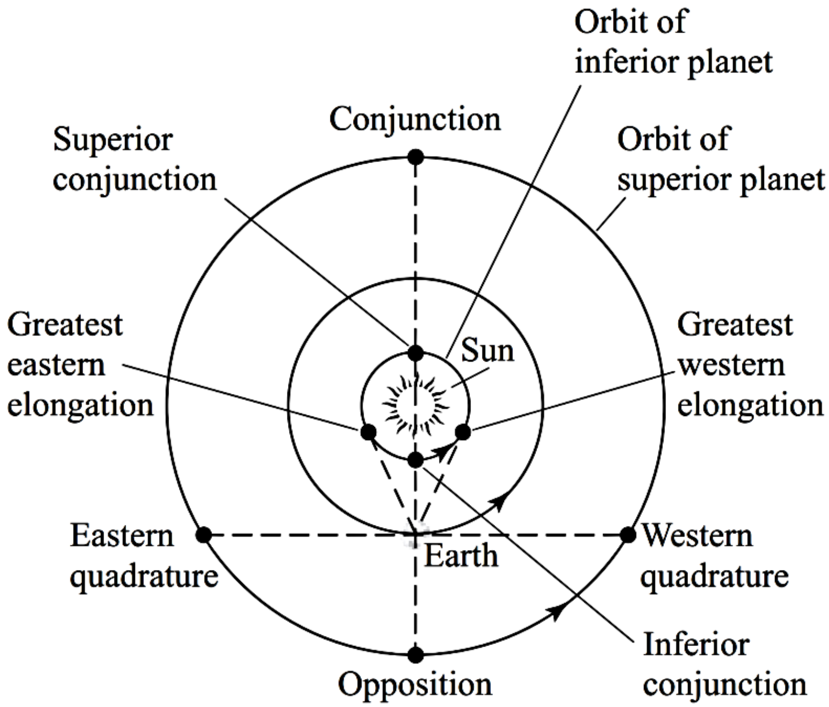
Practice Problem 5

The quasar SDSS 1030+0524 produces a hydrogen emission line of wavelength $\lambda_{rest} = 121.6\,nm$. On Earth, this emission line is observed to have a wavelength of $\lambda_{obs} = 885.2\,nm$:

1. What is the redshift of this quasar?
2. Determine the radial velocity of the quasar.
3. Determine the distance of this quasar.

Planetary motion

The *apparent motions* of the planets are quite complicated, partly because they reflect the motion of the Earth around the Sun. Normally the planets move eastward (*direct motion*, counterclockwise as seen from the Northern hemisphere) when compared with the stars.



The fact that Mercury and Venus are never seen more than 28° and 47°, respectively, east or west of the Sun clearly shows that their orbits are located inside the orbit of Earth. These planets are referred to as **inferior planets**, and their maximum angular separations east or west of the Sun are known as greatest eastern elongation and greatest western. elongation, respectively (see Figure above).

Opposition occurs when the Earth lies between the Sun and the superior planet. That is, the Sun and planet are 180° apart on the celestial sphere as seen from the Earth. **Conjunction** occurs when the Sun lies between the Earth and the superior planet. That is, the Sun and planet are 0° apart as seen from the Earth. **Quadrature** occurs when the Sun and the superior planet are 90° apart as seen from the Earth. The quadrature can be either eastern, when the planet appears 90° east of the Sun on the sky, or western, when the planet appears 90° west of the Sun.

Inferior conjunction occurs when the inferior planet lies between the Earth and the Sun. **Superior conjunction** occurs when the Sun lies between the Earth and the inferior planet.

The relative orbital motions of Earth and the other planets mean that the time interval between successive oppositions or conjunctions can differ significantly from the amount of time necessary to make one complete orbit relative to the background stars. The former time interval (between oppositions) is known as the **synodic period** (S), and the latter time interval (measured relative to the background stars) is referred to as the **sidereal period** (P). The relationship between the two periods is given by

$$\frac{1}{S} = \left|\frac{1}{T_p} - \frac{1}{T_E}\right| \qquad \text{Equation 23}$$

Where T_p is the period of rotation of a random planet, T_E is the period pf Earth (1 year), and S is the synodic period (or period between two successive same situations of the two planet).

Practice Problem 6

The time interval between two successive oppositions of Mars is $779.9\ days$. Calculate the semimajor axis of Mars' orbit.

Practice Problem 7

We are making an observation on 1st day of February 2022. We observe that Mars is in opposition; at the same time, we see that Jupiter is also in western quadrature:

a) Determine the date of the next conjunction of Mars.
b) Determine the date of the next opposition of Jupiter.
c) Find the distance of Mars and Jupiter on February 1, 2022.
d) Determine the date of the next opposition of Mars and Jupiter.
e) What is the angle of Earth-Mars-Jupiter on the next opposition of Jupiter?
f) Discuss the situation when all three planets are on one side of the Sun on a line. This mean that Mars and Jupiter are going to be in opposition with Earth at the same time. When do you think this happens?

Spherical Astronomy

We have seen that the observer who views the heavens at night gets the impression that they are at the centre of a great hemisphere onto which the heavenly bodies are projected. The moon, planets and stars seem to lie on this celestial hemisphere, their directions defined by the positions they have on its surface. For many astronomical purposes the distances are irrelevant so that the radius of the sphere can be chosen at will. The description of the positions of bodies on it, considering positional changes with time, necessarily involves the use of special coordinate and timekeeping systems. The relationship between positions of bodies requires a knowledge of the geometry of the sphere. This branch of astronomy, called **spherical astronomy**, is in one sense the oldest branch of the subject, its foundations dating back at least 4000 years. Its subject matter is still essential and never more so than today, when the problem arises of observing or calculating the position of an artificial satellite or interplanetary probe. We, therefore, begin by considering the geometry of the sphere.

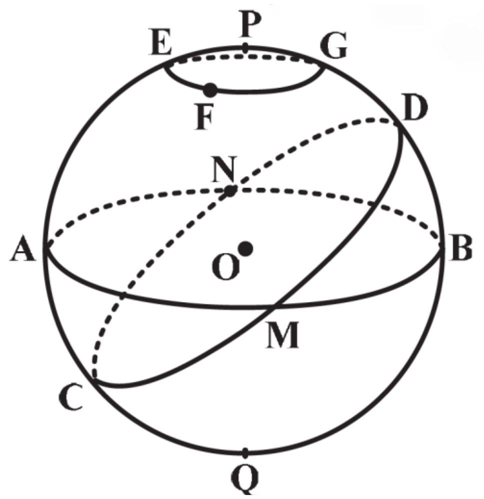
Spherical geometry

The geometry of the sphere is made up of great circles, small circles and arcs of these figures. Distances along great circles are often measured as angles since, for convenience, the radius of the sphere is made unity.

A **great circle** is defined to be the intersection with the sphere of a plane containing the centre of the sphere. Since the centre is equidistant from all points on the sphere, the figure of intersection must be a circle. If the plane does not contain the centre of the sphere, its intersection with the sphere is a **small circle**.

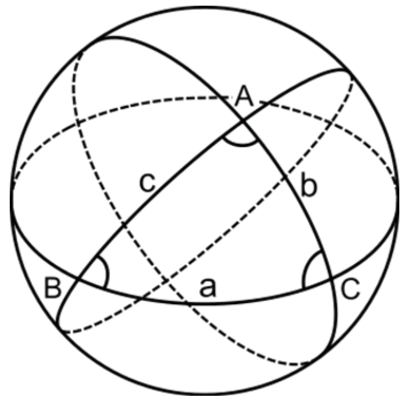
We can draw infinite circles on a sphere, some may have the radius of the sphere (great circles) and others will have smaller radius (small circles).

In the figure on the right *ANBM*, *CNDM* and *APBQ* are all great circles, while *EFG* is a small circle.



If three great circles intersect one another so that a closed figure is formed by three arcs of the great circles, it is called a **spherical triangle** if it possesses the following properties:

1. Any two sides are together greater than the third side.
2. The sum of the three angles is greater than 180°.
3. Each spherical angle is less than 180°.

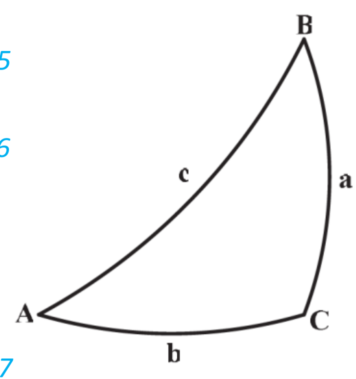


The area of the spherical triangle can be found by the equation:

$$S_{ABC} = (A + B + C - \pi)R^2 \qquad \text{Equation 24}$$

Just as the formulas of plane trigonometry can be used to perform calculations in plane geometry, special trigonometrical formulas for use in spherical geometry can be established. There are many such formulas but four are more often used than any of the others. They are the relations between the sides and angles of a spherical triangle and are invaluable in solving the problems that arise in spherical astronomy.

ABC is a spherical triangle with sides *AB*, *BC* and *CA* of lengths *c*, *a* and *b*, respectively and with angles $\widehat{CAB}$, $\widehat{ABC}$ and $\widehat{BCA}$, hereafter referred to as angles *A*, *B* and *C* respectively. The four formulas are:



Sine formula:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad \text{Equation 25}$$

Cosine formula:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \qquad \text{Equation 26}$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Polar formula

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a \qquad \text{Equation 27}$$

$$-\cos B = \cos B \cos C - \sin A \sin C \cos b$$

$$-\cos C = \cos A \cos B - \sin A \sin B \cos c$$

Four-parts formula

$$\cos a \cos C = \sin a \cot b - \sin C \cot B \qquad \text{Equation 28}$$

$$\cos(inner\ side) \cos(inner\ angle) = \sin(inner\ side) \cot(other\ side) - \sin(inner\ angle) \cot(other\ angle)$$

Practice Problem 8

Solve completely the spherical triangle ABC, and find the area of the triangles: (assume $R = 1$)

a) $a = 34°\,46', b = 27°\,22', C = 72°\,31'$

b) $b = 98°\,18', C = 24°\,49', A = 68°\,36'$

c) $a = 14°\,03', b = 53°\,32', c = 124°\,14'$

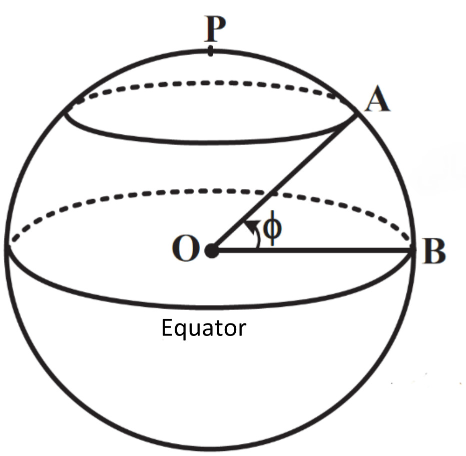
d) $A = 23°\,32', B = 102°\,38', C = 34°\,44'$

Position on the Earth's surface

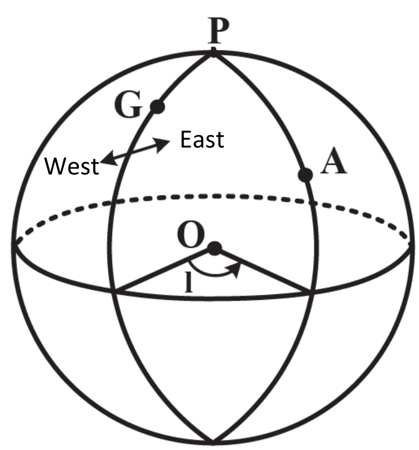
To illustrate these concepts, we consider the Earth. Geographers have already shown us how to set up a coordinate system on a sphere; the system of **latitude** and **longitude** provides a coordinate system on the surface of the (approximately) spherical Earth. On the Earth, the north and south poles represent the points where the Earth's rotation axis passes through the Earth's surface.

The **equator** is the great circle midway between the north and south pole, dividing the Earth's surface into a northern hemisphere and a southern hemisphere. The latitude of a point on the Earth's surface is its angular distance from the equator, measured along a great circle perpendicular to the Earth's equator. Latitude is measured in degrees, arcminutes, and arcseconds, as is longitude. Thus, the use of latitude and longitude doesn't require knowing the size of the Earth in kilometers or any other unit of length.

Latitude: φ



Longitude: l



The longitude may be expressed in angular measure or in time units related to each other by the table on the right.

$360° = 24^h$	
$1° = 4^m$	$1^h = 15°$
$1' = 4^s$	$1^m = 15'$
$1'' = (1/15)^s$	$1^s = 15''$

Practice Problem 9

Two cities A and B on the same parallel of latitude $\varphi = 43°\,39'N$ are $127°\,22'$apart in longitude. Calculate in kilometers:

a) their distance apart along the parallel. (the small circle between the cities with same latitude)

b) the great circle distance *AB*.

c) Determine the highest latitude of the great circle between two cities.

The horizontal (alt-azimuth) system

It is convenient to imagine a sphere at great distance ("infinity") upon which all stars lie. This is called the *celestial sphere*. The positions of stars on this sphere may be specified with two angles, analogous to the way latitude and longitude specify a position on the earth's surface. This celestial sphere is an artificial construction; stars are *not* all at the same distance. Stars in our own Galaxy range in distance from 4 light years to more than 50000 light years from the Earth. Nevertheless, the concept of celestial sphere is useful for charting the sky as one sees it.

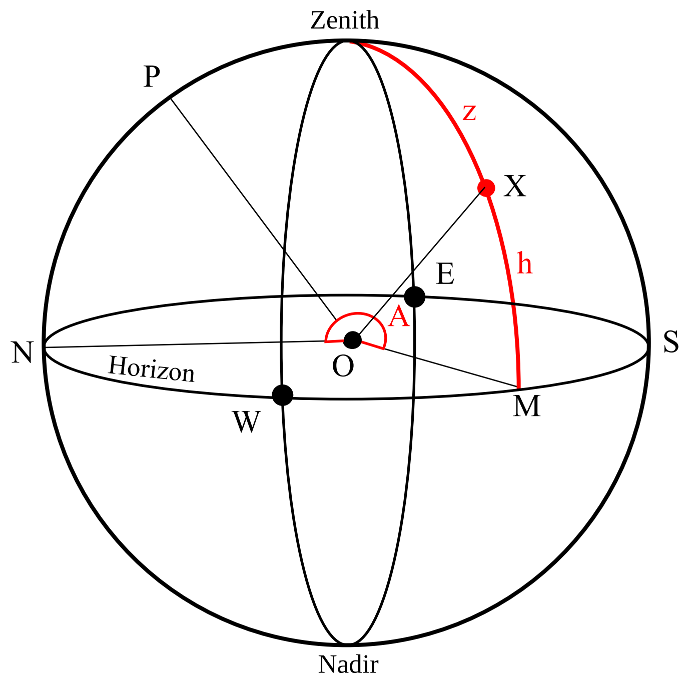
One such coordinate system on the celestial sphere is based on an observer's horizon, and hence is called the **horizon coordinate system**. In this system, the latitude-like coordinate is the **altitude**, defined as the angle of a celestial object above the horizon circle. The zenith (the point directly overhead) is at an altitude of 90°. Points on the horizon circle are at an altitude of 0°. The nadir is at an altitude of -90°, but in practice, negative altitudes are seldom used, since they represent objects that are hidden by the Earth. The longitude-like coordinate in the horizon coordinate system is called the **azimuth**.

In the figure on the right, X, is the position of the star. The arc of $\widehat{XM} = h$,where h is the altitude. Therefore, the zenith distance is $\widehat{XZ} = 90 - h = z$.

The azimuth of this star is the red angle shown in the figure: $A = 360 - \widehat{NOM}$.

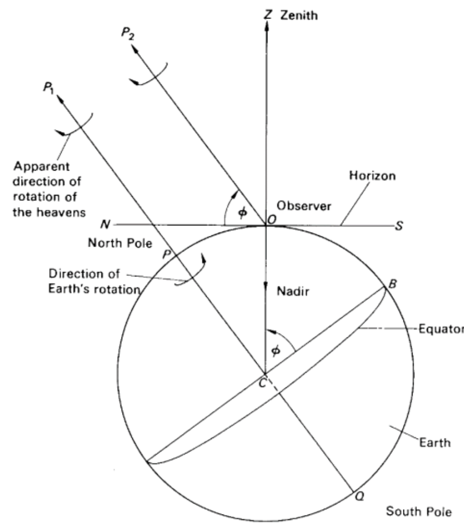
Azimuth is usually expressed from North to East. But if the star is located in the western hemisphere (like the star in the figure), we can express the azimuth from North to West:

$A = \widehat{NOM}\ W$



For any point on the celestial sphere, half a great circle can be drawn from the zenith, through the point in question, to the nadir. The half-circle that runs through the north point on the horizon circle acts as the “prime meridian” in the horizon coordinate system. The azimuth is measured in degrees running from north through east. An object due north of an observer has an azimuth of 0°, an object due east has an azimuth of 90°, and so forth. If you know the altitude and azimuth of any object in your horizon coordinate system, you know where to point your telescope to see it.

If we consider the figure on the right for an observer in a particular latitude of φ, the direction of rotation of the Earth is P_1, and since north celestial pole (NCP) is in distant, P_2 will be the direction for that the person will see the north celestial pole. It is depicted that the altitude of the pole is equal to the latitude of the observer.



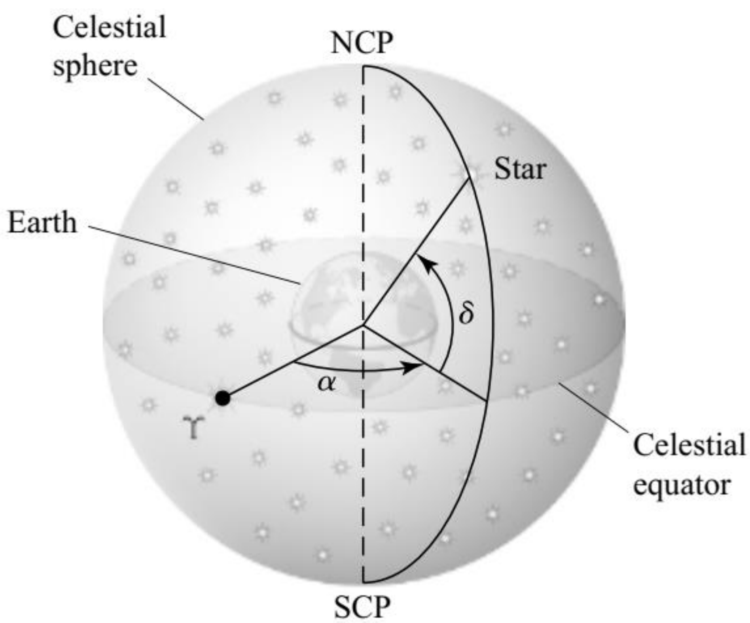
One shortcoming of the horizon coordinate system is that every observer on Earth has a different, unique horizon and hence has a different, unique horizon coordinate system. A star that is near the zenith (altitude $\approx 90°$) for an observer in Buenos Aires will be near the nadir (altitude $\approx -90°$) for an observer in the antipodal city of Shanghai. To describe positions of objects on the celestial sphere, it is useful to have a coordinate system that all astronomers, regardless of location, can agree on, just as geographers all agree to use latitude and longitude to describe positions on the Earth.

The equatorial system

To build a coordinate system useful for all Earthlings, we start by projecting the Earth’s poles and equator outward onto the celestial sphere. The Earth’s rotation axis, which passes through the north and south poles of the Earth, intersects the celestial sphere at the **north celestial pole** (labeled as NCP) and the **south celestial pole** (labeled as SCP). The north celestial pole is at the zenith for an observer at the Earth’s north pole; more generally, for an observer at a latitude north of the equator, it will be at an altitude of and an azimuth of 0°. The projection of the Earth’s equator onto the celestial sphere is called the **celestial equator**. The celestial equator passes through the zenith for an observer on the Earth’s equator.

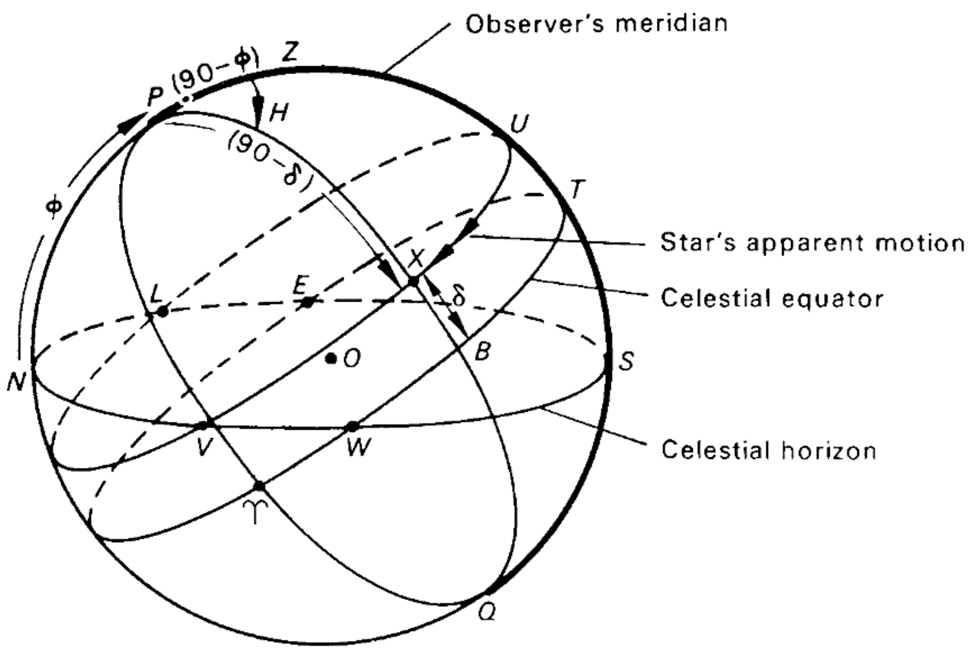
On the Earth's surface, a point's latitude is its angular distance north or south of the equator. Similarly, on the celestial sphere, a point's **declination** (δ) is its angular distance north or south of the celestial equator. For points north of the celestial equator, the declination is positive ($0° < \boldsymbol{\delta} \leq 90°$), and for points south of the celestial equator, the declination is negative ($-90° \leq \boldsymbol{\delta} < 0°$).

Right ascension α is analogous to longitude and is measured eastward along the celestial equator from the vernal equinox (♈) to its intersection with the object's hour circle (the great circle passing through the object being considered and through the north celestial pole). Right ascension is traditionally measured in hours, minutes, and seconds. The coordinates of right ascension and declination are also indicated in figure below. Since the equatorial coordinate system is based on the celestial equator and the vernal equinox, changes in the latitude and longitude of the observer do not affect the values of right ascension and declination. Values of α and δ are similarly unaffected by the annual motion of Earth around the Sun.



We used a unique point to define Azimuth angle in horizontal coordinates, that specific point is North. Having that in mind, we also know that the NCP is pointing towards the North. We can draw both coordinates on a sphere for an observer.

If we sketch both coordinates on a single sphere, then the celestial equator intersects the horizon circle in two points *West* and *East*. Points *P* and *Z* are the poles of the celestial equator and the horizon respectively. But *W* lies on both these great circles so that *W* is 90° from the points *P* and *Z*. Hence, *W* is a pole on the great circle *Z P N* and must, therefore, be 90° from all points on it—in particular from *N* and *S*. Hence, it is the west point. By a similar argument *E* is the east point. Any great semicircle through *P* and *Q* is called a **meridian**. The meridian through the celestial object *X* is the great semicircle *PXBQ* cutting the celestial equator in *B*



In particular, the meridian *PZTSQ*, indicated because of its importance by a heavier line, is the **observer's meridian**. An observer viewing the sky will note that all natural objects rise in the east, climbing in altitude until they **transit** across the observer's meridian then decrease in altitude until they set in the west. A star, in fact, will follow a small circle parallel to the celestial equator in the arrow's direction. Such a circle (*UXV* in the diagram) is called a **parallel of declination** and provides us with one of the two coordinates in the equatorial system. The **declination**, δ, of the star is the angular distance in degrees of the star from the equator along the meridian through the star. It is measured north and south of the equator from 0° to 90°, being taken to be positive when north. The declination of the celestial object is thus analogous to the latitude of a place on the Earth's surface, and indeed the latitude of any point on the surface of the Earth when a star is in its zenith is equal to the star's declination.

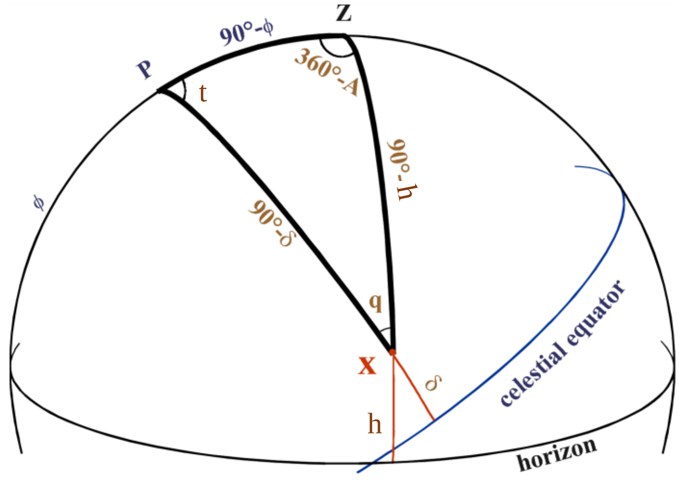
A quantity called the **north polar distance** of the object (X in figure) is often used. It is the arc PX.

Obviously,

$$north\ polar\ distance = 90° - declination$$

It is to be noted that the north polar distance can exceed 90°. The star, then, transits at U, sets at V, rises at L and transits again after one rotation of the Earth. The second coordinate recognizes this. The angle ZPX is called the **hour angle**, H, of the star and is measured from the observer's meridian westwards (for both north and south hemisphere observers) to the meridian through the star from 0^h to 24^h or from $0°$ to $360°$. Consequently, the hour angle increases by 24^h each sidereal day for a star.

Having both coordinates on the sphere, using Zenith, North celestial pole, and the star (three points) we are able to create a spherical triangle (figure below). We need to use the spherical trigonometry to solve any spherical triangle.



A common problem in spherical astronomy is to obtain a star's coordinates in one system, given the coordinates in another system. The observer's latitude is usually known. For example, we may want to calculate the hour angle of t and declination δ of a body when its azimuth (east of north) and altitude are A and h. Assume the observer has a latitude φ.

We start by writing cosine formula:

$$\cos PX = \cos PZ \cos ZX + \sin PZ \sin ZX \cos PZX \quad \rightarrow \quad \sin\delta = \sin\varphi \sin h + \cos\varphi \cos h \cos A$$

By using the cosine formula again:

$$\cos ZX = \cos PZ \cos PX + \sin PZ \sin PX \cos ZPX \quad \rightarrow \quad \sin a = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos t$$

You could also use four-parts or sine law to solve the spherical triangle. Based on the known parameters in the triangle, you should decide which formulas to use in order to solve the triangle.

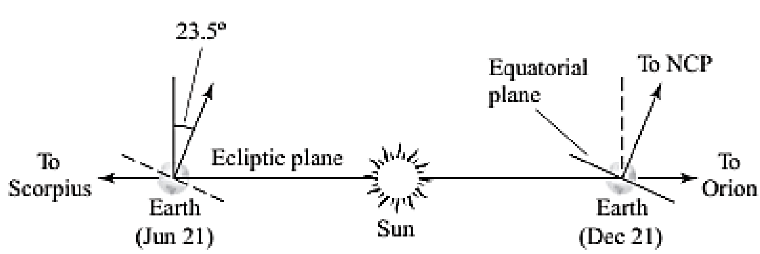
Practice Problem 10

An observer is tracking Rigel $\delta_R = -8°\,12'$, $\alpha_R = 5^h\,14^m$ in Toronto ($\varphi_{Toronto} = 43.65°N$)

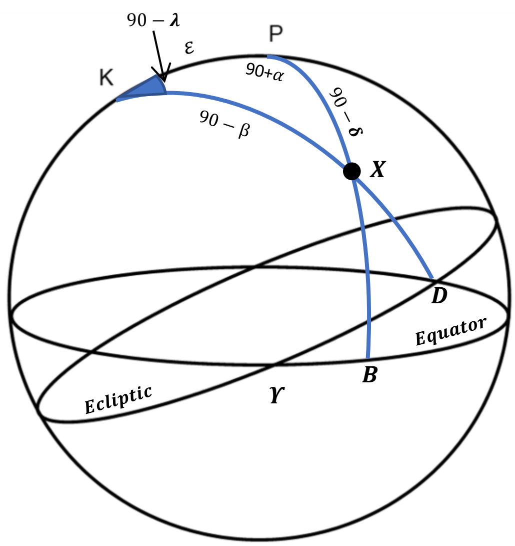
a) What is the maximum altitude of this star in Toronto's sky?
b) What is the star's Azimuth at rise? What about the setting Azimuth?
c) What is the star's Azimuth and Hour angle when its altitude is $\mathrm{h} = 8°$?
d) What is the star's altitude and Azimuth when $\mathrm{t} = 1^h 53^m$?
e) What angle does the star's path make with horizon at rise/set?

The ecliptic system

The orbital plane of the Earth, the *ecliptic*, is the reference plane of another important coordinate frame. The ecliptic can also be defined as the great circle on the celestial sphere described by the Sun in the course of one year. This frame is used mainly for planets and other bodies of the solar system. The orientation of the Earth's equatorial plane remains invariant, unaffected by its annual motion. In spring, the Sun appears to move from the southern hemisphere to the northern one. The time of this remarkable event as well as the direction to the Sun at that moment are called the *vernal equinox*. At the vernal equinox, the Sun's right ascension and declination are zero.



The two quantities specifying the position of an object on the celestial sphere in this system are ecliptic longitude and ecliptic latitude. In figure below a great circle arc through the pole of the ecliptic *K* and the celestial object *X* meets the ecliptic in the point *D*. Then the **ecliptic longitude**, λ, is the angle between and *D*, measured from 0° to 360° along the ecliptic in the eastwards direction, that is in the direction in which right ascension increases. The **ecliptic latitude**, β, is measured from *D* to *X* along the great circle arc *DX*, being measured from 0° to 90° north or south of the ecliptic. It should be noted that the north pole of the ecliptic, *K*, lies in the hemisphere containing the north celestial pole. It should also be noted that ecliptic latitude and longitude are often referred to as **celestial latitude** and **longitude**.



Let's assume the equatorial coordinates of a star is known, and we want to determine its ecliptic coordinates. This means α and δ is given. Using the spherical triangle above, we can use cosine law:

$$\cos(90-\beta) = \cos\varepsilon\cos(90-\delta) + \sin\varepsilon\sin(90-\delta)\cos(90+\alpha)$$

$$\sin\beta = \cos\varepsilon\sin\delta - \sin\varepsilon\cos\delta\sin\alpha$$

By using the cosine formula again:

$$\cos(90-\delta) = \cos\varepsilon\cos(90-\beta) + \sin\varepsilon\sin(90-\beta)\cos(90-\lambda)$$

$$\sin\delta = \cos\varepsilon\sin\beta + \sin\varepsilon\cos\beta\sin\lambda \quad \rightarrow \quad \sin\lambda = \frac{\sin\delta - \cos\varepsilon\sin\beta}{\sin\varepsilon\cos\beta}$$

Practice Problem 11

Determine the ecliptic coordinates of Rigel $\delta_R = -8° \ 12',\ \alpha_R = 5^h \ 14^m$.

Practice Problem 12

Show that the point on the horizon at which a star rises is

$$sin^{-1}(sec\ \varphi\ sin\ \delta)$$

Practice Problem 13

We have the coordinates of Vega $\delta_R = 38° \ 47',\ \alpha_R = 18^h \ 36^m$. A person in Toronto ($\varphi_{Toronto} = 43.65°N$) is observing this star:

1. Determine the hour angle of Vega when it rise/set.
2. What is the Azimuth of rise and set of Vega in Toronto's horizon?
3. Determine its maximum altitude in Toronto.
4. Determine the total time Vega is above horizon.
5. On which date does Vega rise at the same time as the Sun in Toronto?

References

1) Smart, W. M., and Green, Robin Michael, Textbook on Spherical Astronomy, Sixth Edition, Cambridge University Press, Cambridge, 1977.

2) Böhm-Vitense, Erika, *Introduction to Stellar Astrophysics, Volume 1: Basic Stellar Observations and Data*, Cambridge University Press, Cambridge, 1989a.

3) Böhm-Vitense, Erika, Introduction to Stellar Astrophysics, Volume 2: Stellar Atmospheres, Cambridge University Press, Cambridge, 1989b.

4) Harwit, Martin, *Astrophysical Concepts*, Third Edition, Springer-Verlag, New York, 1998.

5) Ryden, Barbara Sue, and Bradley M. Peterson. *Foundations of Astrophysics*. Cambridge University Press, 2021.

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7) Carroll, Bradley W., and Dale A. Ostlie. *An Introduction to Modern Astrophysics*. Cambridge University Press, 2018.

8) Roy, Archie E., and David Clarke. *Astronomy: Principles and Practice*. CRC Press, 2018.

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10) Schneider, Stephen E., and Thomas Arny. *Pathways to Astronomy*. McGraw-Hill Education, 2021.

Syllabus of International Astronomy and Astrophysics Olympiad (IOAA)

General Notes:

1. Extensive contents in basic astronomical concepts are required in theoretical and practical problems.
2. Basic concepts in physics and mathematics at high school level are required in solving the problems. Standard solutions should not involve use of calculus and/or the use of complex numbers and/or solving differential equations.
3. Astronomical software packages may be used in practical and observational problems. The contestants will be informed the list of software packages to be used at least 3 months in advance. The chosen software packages should be preferably freeware or low-cost ones enabling all countries to obtain them easily for practice purpose. The chosen software should preferably be available on multiple OSs (Windows / Unix / GNU-Linux / Mac).
4. Concepts and phenomena not included in the Syllabus may be used in questions but sufficient information must be given in the questions so that contestants without previous knowledge of these topics would not be at a disadvantage.
5. Sophisticated practical equipment likely to be unfamiliar to the candidates should not dominate a problem. If such devices are used in the questions, sufficient information must be provided. In such cases, students should be given opportunity to familiarise themselves with the equipment before the exam.
6. The original text of the problems must use SI units, wherever applicable. Participants will be expected to give appropriate units in their answers and should be familiar with the idea of correct rounding off and expressing the final result(s) and error(s) with the correct number of significant digits.

Theoretical Part

Symbol (Q) is attached to some topics in the list, meaning "qualitative understanding only". Quantitative reasoning / proficiency in these topics is not expected.

The following theoretical contents are proposed for the contestants.

Basic Astrophysics

Contents	Topics
Celestial Mechanics	Newton's Laws of Gravitation, Kepler's Laws for circular and non-circular orbits, Roche limit, barycentre, 2-body problem, Lagrange points
Electromagnetic Theory & Quantum Physics	Electromagnetic spectrum, Radiation Laws, Blackbody radiation
Thermodynamics	Thermodynamic equilibrium, Ideal gas, Energy transfer
Spectroscopy and Atomic Physics	Absorption, Emission, Scattering, Spectra of Celestial objects, Doppler effect, Line formation, Continuum spectra, Splitting and Broadening of spectral lines, polarisation
Nuclear Physics	Basic concepts including structure of atom, Mass defect and binding energy Radioactivity, Neutrinos (Q)

Coordinates and Times

Contents	Topics
Celestial Sphere	Spherical trigonometry, Celestial coordinates and their applications, Equinox and Solstice, Circumpolar stars, Constellations and Zodiac (Note: azimuth is measured in the range 0° to 360° starting from N and increasing towards E unless stated otherwise.)
Concept of Time	Solar time, Sidereal time, Julian date, Heliocentric Julian date, Time zone, Universal Time, Local Mean Time, Different definitions of "year", Equation of time

Solar System

Contents	Topics
The Sun	Solar structure, Solar surface activities, Solar rotation, Solar radiation and Solar constant, Solar neutrinos (Q), Sun-Earth relations, Role of magnetic fields (Q), Solar wind and radiation pressure, Heliosphere (Q), Magnetosphere (Q)
The Solar System	Earth-Moon System, precession, nutation, libration, Formation and evolution of the Solar System (Q), Structure and components of the Solar System (Q), Structure and orbits of the Solar System objects, Sidereal and Synodic periods, Retrograde motion, Outer reaches of the solar system (Q)
Space Exploration	Satellite trajectories and transfers, Human exploration of the Solar System (Q), planetary missions (Q), Sling-shot effect of gravity, Space-based instruments (Q)
Phenomena	Tides, Seasons, Factors influencing climate (Q), Eclipses, Aurorae and space weather (Q), Meteor Showers

Stars

Contents	Topics
Stellar Properties	Methods of Distance determination, Radiation, Luminosity and magnitude, Color indices and temperature, Determination of radii and masses, Stellar motion, Irregular and regular stellar variabilities – broad classification & properties, Cepheids and period-luminosity relation, Physics of pulsation (Q)
Stellar Interior and Atmospheres	Stellar equilibrium, Stellar nucleosynthesis, Energy transportation (Q), Boundary conditions, Stellar atmospheres and atmospheric spectra
Stellar Evolution	Stellar formation, Hertzsprung-Russell diagram, Pre-Main Sequence, Main Sequence, Post-Main Sequence stars, supernovae, planetary nebulae, End states of stars

Stellar Systems

Contents	Topics
Binary Star Systems	Different types of binary stars, Mass determination in binary star systems, Light and radial velocity curves of eclipsing binary systems, Doppler shifts in binary systems, interacting binaries, peculiar binary systems
Exoplanets	Techniques used to detect exoplanets, Habitable zone, Classes of exoplanets (Q), Spectral signatures of possible life (Q)
Star Clusters	Classification and Structure, Mass, age, luminosity and distance determination
Milky Way Galaxy	Structure and composition, Rotation, Satellites of Milky Way (Q)
Interstellar Medium	Gas (Q), dust (Q), HII regions, 21cm radiation, nebulae (Q), interstellar absorption, dispersion measure, Faraday rotation
Galaxies	Classifications based on structure, composition and activity, Mass, luminosity and distance determination, Rotation curves
Accretion Processes	Basic concepts (spherical and disc accretion) (Q), Eddington luminosity

Cosmology

Contents	Topics
Elementary Cosmology	Expanding Universe and Hubble's Law, Cluster of galaxies, Dark matter, Dark energy (Q), Gravitational lensing, Cosmic Microwave Background Radiation, Big Bang (Q), Alternative models of the Universe (Q), Large scale structure (Q), Distance measurement at cosmological scale, cosmological redshift

Instrumentation and Space Technologies

Contents	Topics
Multi-wavelength Astronomy	Observations in radio, microwave, infrared, visible, ultraviolet, X-ray, and gamma-ray wavelength bands, Earth's atmospheric effects, Artificial light and EM pollution
Instrumentation	Telescopes and detectors (e.g. charge-coupled devices, photometers, spectrographs), Magnification, Focal length, Focal ratio, resolving and light-gathering powers of telescopes, Geometric model of two element interferometer, Aperture synthesis, Adaptive optics, photometry, astrometry

Practical Part

This part consists of 2 sections: observations and data analysis sections. The theoretical part of the Syllabus provides the basis for all problems in the practical part.

The observations section focuses on contestant's experience in:

1. naked-eye observations,
2. usage of sky maps and catalogues (note: any stars referred to by name rather than Bayer designation or catalogue number must be on the list of IAU-approved star names; knowledge of the whole list should not be required).
3. application of coordinate systems in the sky, magnitude estimation, estimation of angular separation
4. usage of basic astronomical instruments-telescopes and various detectors for observations but enough instructions must be provided to the contestants. Observational objects may be from real sources in the sky or imitated sources in the laboratory. Computer simulations may be used in the problems but sufficient instructions must be provided to the contestants.

The data analysis section focuses on the calculation and analysis of the astronomical data provided in the problems. Additional requirements are as follows:

1. Proper identification of error sources, calculation of errors, and estimation of their influence on the final results.
2. Proper use of graph papers with different scales, e.g., polar and logarithmic papers. Transformation of the data to get a linear plot and finding "Best Fit" line approximately.
3. Basic statistical analysis of the observational data.
4. Knowledge of the most common experimental techniques for measuring physical quantities mentioned in Part A.