CAAO-Jr 2024

Canadian Astronomy and Astrophysics Olympiad

May 26, 2024

Full Name: _____

Date of Birth: _____

Grade: _____

School: _____

Instructions:

- This exam comprises **6** problems.
- Read the instructions for each question carefully.
- Please provide legible handwritten answers on separate pieces of paper.
- Show all your work and justify your answers clearly and concisely.

Note: This exam is worth **115** points.

1 Signal Delay Astronomy (20 points)

An asteroid has a circular motion around the Sun on the ecliptic plane. A spaceship lands on this asteroid and sends a signal to the mission control center on Earth, and whenever mission control receives a signal, they immediately answer back with another signal to the spaceship. When this Asteroid is in quadrature, we know that the time interval for a signal to go from the spaceship to the Earth and from Earth to the spaceship again is $\Delta t = 775.6s$ longer than when the asteroid is in opposition. Assume Earth's orbit is also circular.

(a) Determine the Asteroid's distance from Sun in terms of Δt, r_{ES} (Earth-Sun distance), and c (light speed).

(b) Find the distance in terms of AU and km. ($r_{ES} = 1.5 \times 10^{11} m = 1AU$)

2 Stellar Temperature (10 points)

If the angular diameter of a star is 0.01 *arcseconds* and its luminosity is 4.5×10^{-8} W/m^2, what is the effective temperature of the star?

3 Observational Astronomy (15 points)

An observer in city A with coordinates $\phi_A = 50^\circ\ N$ and $l_A = 67^\circ\ E$ is observing a star with right ascension of $\alpha = 63^\circ$ exactly on the horizon while doing its lower culmination.

(a) Determine the declination of this star.

(b) What is the star's zenith distance doing its upper culmination?

(c) If $\epsilon = 23.5^\circ$, determine celestial latitude and longitude of this star.

(d) Find the sidereal time at city A and Greenwich at the time of observation.

4 Star Visibility in Toronto (20 points)

You are situated at the center of a street in Toronto. This street is oriented north-south, has a width of 30 meters, and is lined with buildings that all stand 20 meters high. Let's assume that this street extends infinitely in both directions. Determine for how long you can observe a star with a declination of $\delta = 30^\circ$ from the point where you are standing.

5 Apparent Magnitudes: Earth and Venus (10 points)

We have an observer on Earth looking at the planet Venus and an observer on Venus looking at Earth. The apparent magnitude recorded by the observer on Venus for Earth is denoted as m_E, and the apparent magnitude recorded by the observer on Earth for Venus is denoted as m_V. Determine the difference in apparent magnitudes $(m_E - m_V)$ when the distance between Earth and Venus is 0.5 AU.

6 The Wandering Earth (40 points)

In the scientific novel *The Wandering Earth*, humans are on the brink of extinction as the Sun expands and will soon engulf Earth. This led to the construction of thousands of fusion-based engines to thrust the planet off its orbit, away from the Sun. In this question, several aspects of this journey are investigated.

1. First, Earth's rotation about its own axis must be stopped. This is done by simultaneously firing two super-dense blocks, each with mass 1/100 that of Earth ($m = M/100$), at two diametrically opposite points on the equator. Assuming Earth has uniform density, what is the speed (relative to the surface) at which each block should be launched such that Earth stops rotating afterward, without affecting Earth's linear speed? Assume both blocks are launched tangent to the surface and in the plane of the equator.

2. The engine is powered by the fusion reaction below (this is also how the Sun produces energy). Assuming 0.01% of Earth's mass is hydrogen, and the energy is 100% converted, is it enough for Earth to escape the Sun's gravitational pull?

$$4\ {}^{1}\mathrm{H}^{+} + 2\ \mathrm{e}^{-} \rightarrow\ {}^{4}\mathrm{He}^{2+}$$

   Data: $m_{\mathrm{H}^+} = 1.007276\ \mathrm{u}, m_{\mathrm{e}^-} = 0.000549\ \mathrm{u}, m_{\mathrm{He}^{2+}} = 4.001506\ \mathrm{u}, 1\ \mathrm{u} = 1.66 \times 10^{-24}\ \mathrm{g}$

3. To save energy, Earth is going to undergo gravity assist near Jupiter. To do this, Earth's speed is first increased in an instant (the direction of the velocity stays the same).
   (a) What is increase in speed needed such that in the new orbit, Earth barely reaches Jupiter's orbit at apoapsis?
   (b) What is the angle between Earth and Jupiter, as seen from the Sun, when Earth's speed is first increased? Note that Jupiter and Earth should meet when Earth reaches apoapsis of the new orbit for the first time.
   (c) What is the increase in energy of Earth caused by the collision? Assume that $M_{\mathrm{J}} \gg M$, and gravity assist is effectively an elastic collision.

4. When Earth is near Jupiter, since it is much farther away from the Sun, its temperature will decrease. Humanity has thus made an underground city for insulation from the cold weather above ground.
   (a) What is the new surface temperature of Earth?
   (b) If the humans stay 1000 km underground, and the thermal conductivity of the crust is 3 W/m/K, what's the heating power needed to maintain a comfy 20 °C? Note: normally, 1000 km underground will put you in lava, but all of that has turned into crust due to the cold temperature.