CAAO 2025 Junior

Canadian Astronomy and Astrophysics Olympiad

May 24, 2025

Full Name: _____

Date of Birth: _____

Grade: _____

School: _____

Instructions:

- This exam comprises **6** problems.
- Read the instructions for each question carefully.
- Please provide legible handwritten answers on separate pieces of paper.
- Show all your work and justify your answers clearly and concisely.

Note: This exam is worth **170** points.

1 Exoplanetary System Analysis (20 points)

In the table below, data is provided for seven planets in an exoplanetary system. However, information for one of the planets is missing. Using the patterns and relationships observed in the data for the known planets:

Planet n	Semi-Major Axis (AU)	Period (Years)	Eccentricity	Mass ($M_\oplus$)
1	1.10	1.63	0.215	0.165
2	1.90	3.70	0.027	0.815
3	3.50	9.26	0.017	1.500
4	6.70	24.53	0.095	0.207
5	25.80	185.67	0.050	419.00
6	51.50	522.67	0.077	95.20
7	102.70	1471.87	0.065	14.50

Table 1: Known planet data in the exoplanetary system

Questions

(a) Determine a relation to calculate the semi-major axis of the planets based on their planet number n, where a_n is the semi-major axis.

(b) Which planet is missing? Determine its semi-major axis and period.

(c) Determine the mass of the central star of this planetary system.

2 Circumpolar stars (25 points)

Circumpolar stars are stars that appear to move around the celestial pole in a complete circle without ever setting below the horizon. Their apparent motion is caused by the rotation of the Earth on its axis. The term "circumpolar" refers to the fact that these stars are always visible above the horizon for an observer located within the corresponding latitude range.

In this problem, we focus on the observation of circumpolar stars at a specific location with a latitude of $\phi = 30^\circ\ N$. An intriguing observation is made in that while the stars rotate around the north celestial pole, certain stars exhibit a *maximum azimuth* before beginning a gradual descent. This phenomenon raises the question of the nature of the constraint governing the azimuth of circumpolar stars at this latitude and the explanation for this behavior.

(a) At latitude 30° N, what determines whether a star is circumpolar? Is it based on the star's azimuth or declination? Clearly state the condition for circumpolarity at this latitude.

(b) Determine the maximum azimuth for a star with declination $\delta = +85^\circ$.

(c) For circumpolar stars observable at this latitude, define the range of declinations that qualify them as circumpolar. Additionally, specify the corresponding limitations on their azimuthal positions throughout their apparent motion.

3 Binary Star System (20 points)

A binary star system consists of two stars, A and B. Both stars are at the same distance from Earth. Star A has an angular diameter of 0.008 arcseconds and emits a flux of $3.0 \times 10^{-8}\,\mathrm{W/m^2}$. Star B appears half as large in angular diameter but is twice as luminous as star A.

(a) Determine the angular diameter of Star B.

(b) Assuming both stars behave like blackbodies, calculate the ratio of their effective temperatures T_B/T_A.

(c) If we cannot resolve these two stars apart, what is the total apparent magnitude of the binary system?

(d) If star A has the same luminosity as our Sun, determine its distance from Earth.

4 Observational Astronomy (25 points)

An observer in the city of Toronto, located at latitude $\phi_T = 43°39'$ N and longitude $\lambda_T = 79°23'$ W, is observing a star exactly on top of his head.

(a) Determine the declination of this star.

(b) What is the star's zenith distance doing its lower culmination?

(c) For how long this star will stay above horizon in Toronto?

5 Apparent Magnitudes: Earth and Asteroid (30 points)

An asteroid is moving in an elliptical orbit around the Sun. Its semi-major axis is $a = 10\,\mathrm{AU}$, and the eccentricity of the orbit is given as $e = 0.5$. What is the difference between its maximum and minimum apparent magnitudes for an observer on Earth?

6 System of Uranus (50 points)

Remember to express your answers in the required dimensions. Some questions may be answered independently of others.

Throughout the problem, we will assume that **all celestial bodies move on circular orbits.** You should use the following values of the mathematical, physical, and astronomical constants:

$$\text{Gravitational constant } G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2},$$

$$\text{Astronomical unit } 1\,\text{AU} = 150 \text{ million km},$$

$$\text{Solar mass } M_\odot = 1.989 \times 10^{30}\,\text{kg},$$

$$1 \text{ rad} = 206265''.$$

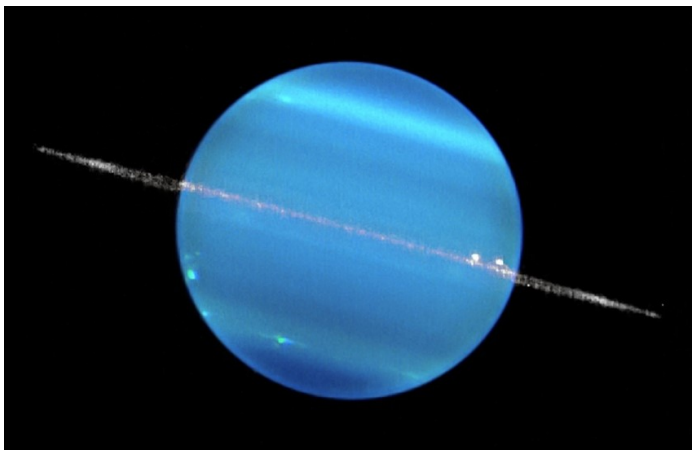

Figure 1: Uranus, as seen by largest modern adaptive optics telescopes.
Source: NASA/JPL/Keck II telescope, Mauna Kea, Hawaii

Uranus – the seventh planet of the Solar system – was discovered by William Herschel in 1781 from his backyard using a self-made telescope. It was soon determined that the synodic period of Uranus (i.e., time between consecutive oppositions) is $S_U = 369.66$ days.

Part I. Orbit and Size

A. The length of Earth's year is $E = 365.25$ days. Determine the orbital period of Uranus T_U. Express the answer in Earth years.

B. What would be the error of determination of Uranus orbital period ΔT_U if the synodic period would be determined with an error of $\Delta S_U = 0.1$ days? Express the answer in Earth years!

In the next questions, please assume the Uranus orbital period to be 84 years. This value may differ from the value obtained in previous questions or from the real value.

C. Determine the semi-major axis of the Uranus orbit a_U in astronomical units and the orbital speed v_U in km/s. The orbital speed of Earth is 29.8 km/s.

D. In opposition, the visible diameter of Uranus is $4.0''$. Determine the diameter of Uranus in kilometers.

Video: Movement of Uranus during several hours near its opposition in 2021.
Source: Observatory of Hertfordshire University

Part II. Radiation

E. The solar constant, that is, the power of solar radiation incident to each square meter outside Earth's atmosphere is $k_Z = 1360\,\mathrm{W/m^2}$. Determine the "solar constant" for Uranus, that is, solar radiation power incident on a surface area unit just outside the atmosphere of Uranus ($\mathrm{W/m^2}$)!

F. How much energy does Uranus receive from the Sun every second (W)? For Uranus diameter, use the value 51,000 km (This value may differ from the value obtained in previous questions or from the real value.).

G. What illumination ($\mathrm{W/m^2}$) does Uranus create on the surface of the Earth, neglecting atmospheric absorption? Assume Uranus is in opposition. Use Uranus albedo of 40% (this means that Uranus scatters back to space 40% of the received solar radiation).

H. Determine the apparent magnitude of Uranus in opposition (in magnitudes), if a 0^m star produces illumination of $5 \times 10^{-9}\,\mathrm{W/m^2}$ on the surface of the Earth. Assume that the illumination by Uranus equals $3 \times 10^{-11}\,\mathrm{W/m^2}$ (This value may differ from the value obtained in previous questions).

Part III. Satellites

The largest satellite of Uranus, Titania, is never observed from Earth to be farther away than 30″ from the center of the planet. The orbital period of Titania is 8d 17h.

I. Determine the semi-major axis a_T of Titania (km), assuming its orbit is circular.

J. Determine the mass of Uranus M_U and express it in units of the Earth mass M_E by comparing the motion of Titania to the motion of the Moon. For your estimates, use the Titania orbital radius value of 436,000 km (Note: this value may differ from the value obtained in previous questions or from the real value) and the following data about the Moon: semi-major axis 384,000 km and orbital period 27.32 days.

It is well-known that the rotation axis of Uranus is almost perpendicular to its orbital plane, and that the orbital planes of the largest Uranus' satellites are close to its equatorial plane, that is, almost perpendicular to the orbital plane of Uranus. In the following, we will assume that the orbital plane of Titania is exactly perpendicular to the orbital plane of Uranus.

Change of orientation of Uranus and its rings as seen from Earth during 45 years.

K. Determine the number of Titania eclipses in one year of Uranus (84 years). The Titania eclipse is an event when the full shadow (umbra) of Uranus is falling on Titania. For the Uranus diameter, use the value 51,000 km, neglect the size of Titania. The Sun as seen from Uranus has an angular diameter of 1.5′.