CANADIAN ASTRONOMY AND ASTROPHYSICS OLYMPIAD

2024 Problems

7 problems Total: 115 points

Problem 1 Solar Eclipse (20 points)

Total solar eclipses are a rare sight. If you miss the one on April 8 this year, then you'll have to wait 20 years before North America gets another one. Two astronomers observe a solar eclipse. The first astronomer saw a perfect annular eclipse with an impact distance $\tau_{1,\min} = 0'$, and the second astronomer saw only a partial eclipse, in which at the maximum phase the edge of the Moon's disk passed through the centre of the Sun's disk. The angular radii of the Sun and Moon and their angular velocities during this period had the following values:

$$\rho_{\text{Sun}} = 16'15'', \qquad \rho_{\text{Moon}} = 15'00'',$$
$$\omega_{\text{Sun}} = 2.46'/\text{hour}, \qquad \omega_{\text{Moon}} = 0.54^\circ/\text{hour}.$$

The radius of the Sun is $R_{\text{Sun}} = 696\,340$ km; take $\pi = 3.142$.

Determine:

1. the minimum value of the impact distance for the second astronomer;
2. the maximum phases of the solar eclipse for both astronomers;
3. the total duration of the eclipse for the second astronomer;
4. the duration of the annular phase for the first astronomer;
5. the area of the visible surface of the Sun not covered by the disk of the Moon at the maximum phase for the first astronomer.

Problem 2 Telescope (8 points)

An astronomer observes Venus using a telescope with an objective of focal length 10.8 m. The angular diameter of Venus near its greatest elongation is 25″ (the diameter of Venus is $D = 12\,100$ km).

Determine:

1. the focal length of the eyepiece with which Venus appears 32′ across;
2. the diameter of the image of the planet on a photographic negative taken at the focus of the telescope;
3. the angular scale of the negative;
4. the linear scale of the negative.

Problem 3 Satellite X (15 points)

Suppose Satellite X, which has a mass of 5000 kg, is in a circular orbit in the direction of increasing longitude above the Earth's equator, at a height of 400 km above the Earth's surface. Suddenly, when Satellite X is at the prime meridian, it is hit by an asteroid of mass 80 kg which is travelling directly towards the Earth's centre at a speed of 5 km/s. The collision is totally inelastic, so the satellite and the asteroid form a single mass after the collision. Any thrusters or propellant that the satellite had are destroyed. Ignoring atmospheric effects, determine whether the satellite will crash into the Earth, and if so, at what latitude and longitude this will occur.

Problem 4 Present Delivery (20 points)

This year, Santa Claus is feeling a bit lazy. Instead of flying around to deliver his presents, he constructs a large cannon at the North Pole and shoots the presents to their intended destinations. For this problem, ignore any effects due to air resistance or the rotation of the Earth. The mass of the present is negligible.

(a) In the future, Santa is considering delivery to other planets. What is the minimum speed at which his cannon must shoot the presents for them to escape the Earth's gravitational pull?

For now, however, Santa's operations are constrained to the Earth. He constructs his cannon such that all presents are shot with x times the speed derived in part (a). For his test run, Santa wants to deliver a present to Daniel, who is somewhere in Houston (30° N, 95° W). He fires the cannon in such a way that the present lands exactly where Daniel is.

Suppose $x = \sqrt{\dfrac{\sqrt{3}-1}{2}}$.

(b) What is the semi-major axis of the orbital path of the present, in terms of $R_\oplus$?

There are actually two possible launch angles Santa can choose. For parts (c) and (d), suppose that Santa chooses the larger launch angle (the one that makes the present go higher).

(c) What is the maximum height the present reaches above the surface of the Earth, in terms of $R_\oplus$?

(d) How long after Santa fires the cannon will Daniel receive his present?

Unfortunately, the cannon isn't powerful enough to deliver presents to everyone on Earth.

(e) What fraction of the Earth's surface will Santa be able to deliver presents to?

Of course, Santa would like to deliver presents to everyone; in particular, his cannon needs to be able to shoot all the way to the South Pole. He modifies his cannon so that $x = \sqrt{6}/3$.

(f) At what angle above the horizontal does Santa need to fire the present so that it lands exactly at the South Pole?

Mass of the Earth: $M_\oplus = 5.972 \times 10^{24}$ kg. Radius of the Earth: $R_\oplus = 6378$ km.

Problem 5 Flight Path Analysis (12 points)

An airplane wants to fly from Vancouver ($\phi = 49.3°$ N, $l = 123.1°$ W) to Paris ($\phi = 48.8°$ N, $l = 2.4°$ E). The plane wants to reach Paris as soon as possible, but first it wants to visit the equator.

(a) Explain the minimal path for this flight. (Your explanation must be based on mathematical derivation or precise reasoning.)

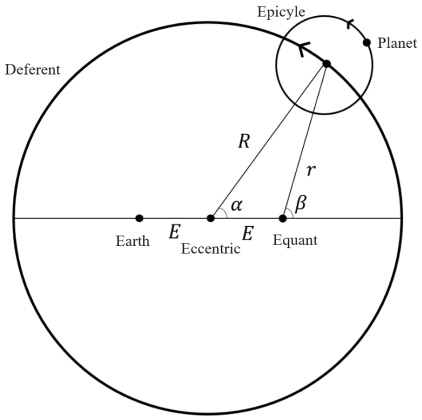
(b) Find the longitude at which the plane meets the equator.

(c) The speed of the plane is 900 km/h. Find the duration of the whole flight.

Problem 6 Epicycle Theory (20 points)

For nearly three hundred years the prevailing model of planetary motion stood unchallenged, until around 150 AD, when the eminent Alexandrian astronomer Claudius Ptolemy observed discrepancies between this model and the most accurate astronomical observations of his time. This inconsistency led Ptolemy to refine the existing framework, an enhancement that became a cornerstone of his seminal work, the *Almagest*.

In his refined model, Ptolemy noted that the epicycle's centre did not maintain a constant velocity as it traversed the deferent. Instead, its speed varied, increasing when nearer to the Earth and decreasing when farther away. While this observation could have necessitated the abandonment of the long-held belief in uniform circular motion, Ptolemy ingeniously proposed a modification. He maintained the concept of circular and uniform motion by introducing the notion that while the epicycle's centre moved in a circular path around one point, its movement was uniform relative to a different point, termed the equant.



The equant point is positioned directly opposite the Earth across the centre of the deferent (the eccentric point), and at the same distance E from the centre as the Earth. The centre of the epicycle maintains a constant distance R from the eccentric point, but it is its angular velocity about the equant point that remains constant, $\Omega = d\beta/dt$. Here α is the angle at the eccentric point, β the angle at the equant, and r the distance from the equant to the centre of the epicycle.

(a) By writing the geometric relationships between the parameters of the figure, prove that

$$\tan\beta = \frac{\sin\alpha}{\cos\alpha - \frac{E}{R}}, \qquad E\sin\beta = R\sin(\beta - \alpha).$$

(b) Since the angular velocity about the equant is constant, the angular velocity about the centre changes with time. Prove that

$$\omega = \frac{d\alpha}{dt} = \frac{\dot{r}}{E\sin\beta}.$$

(c) Find an equation for $\dot{r}$ in terms of r, E, Ω and β, and use it to prove that

$$\omega(t) = \left[1 - \frac{\frac{E}{R}\cos(\Omega t)}{\sqrt{1 - \left(\frac{E}{R}\sin(\Omega t)\right)^2}}\right]\Omega.$$

(d) If $E/R = 0.2$, graph ω/Ω against Ωt. Draw the diagram using five equally spaced points in the range from 0 to 2π.

Problem 7 Sun's Helium Transition (20 points)

The Sun, at the end of its hydrogen-burning phase, reaches a point where its hydrogen-burning core ceases to produce energy. At this time, helium burning does not commence immediately. Instead, due to the loss of nuclear energy generation, the Sun gradually contracts until the core temperature reaches 100 million kelvin (the temperature required for helium burning) and the core density doubles, initiating helium burning.

(a) Using dimensional analysis, determine how the gravitational pressure (the pressure of the overlying column of mass) at the centre of the star, and the gravitational potential energy, depend on G (the gravitational constant), M (the mass of the star) and R (the radius of the star).

(b) Assuming the ideal gas law applies at the Sun's core during this transition, calculate the Sun's radius at the onset of helium burning relative to its current radius.

(c) If the Sun's luminosity changes linearly over this period ($L = L_\odot + \alpha t$), which lasts for 17 million years, calculate the coefficient α.

(d) Derive the equation for the surface temperature as a function of time. Then calculate its numerical value at the onset of helium burning.